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Exciton Condensation Under High Magnetic Field

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The new results in the theory of Bose-Einstein condensation (BEC) of the two-dimensional (2D) magnetoexcitons formed by the high-density electron–hole (*e–h*) pairs created on the semiconductor mono-layer in a strong perpendicular magnetic field are reviewed. One of them is the metastable dielectric liquid phase (MDLP) formed by the 2D magnetoexcitons BEC-ed on the single-particle state with sufficiently large values of the wave vector k, so that its product kl with the magnetic length l equals about $kl \approx 3-4$. This state was revealed in the conditions when the electrons and holes are situated on the lowest Landau levels (LLLs) and the polarizability of the Bose gas was calculated on the base of the Anderson-type coherent excited states. They give rise to correlation energy and to chemical potential displaying a nonmonotonous dependence on the filling factor ν^2 with a relative minimum and with positive compressibility in its vicinity. The influence of the excited Landau levels (ELLs) on the quantum states of the *e–h* system is due to the virtual quantum transitions of particles from the LLLs to ELLs during the Coulomb scattering processes and to their subsequent return back. These quantum transitions were taken into account in the frame of the second order perturbation theory giving rise to an effective Hamiltonian describing the supplementary indirect interactions between the particles lying on the LLLs. This interaction is characterized by a small parameter equal to the ratio r of the magnetoexciton ionization potential $I_{ex}(0)$ to the Landau quantization energy $\hbar\omega_c$. The parameter $r = I_{ex}(0)/\hbar\omega_c$, decreases as $H^{-1/2}$ with the increasing the magnetic field strength H. The supplementary interaction is attractive, making the magnetoexcitons in the Hartree approximation more robust. Nevertheless its exchange, Fock terms as well as the Bogoliubov *u–v* transformation terms give rise to positive, repulsion-type contributions to the chemical potential. The Bose gas of magnetoexcitons with $k = 0$ becomes weakly nonideal when the ELLs are taken into account. The collective elementary excitations of two ground states corresponding to BEC-ed magnetoexcitons forming either a nonideal Bose gas with $k = 0$ or the MDLP with $kl \approx 3-4$ were studied in the frame of the perturbation theory with the infinitesimal parameter $\nu^2(1-\nu^2)$, chosen as a product of the filling factor ν^2 and of the phase space filling factor $(1-\nu^2)$. The collective elementary excitations in both cases consist from the exciton and plasmon branches. Due to the presence of the condensate there are energy and quasi-energy branches. The self-energy parts containing the unknown frequency in denominators increase the degree of the dispersion equations and give rise to mixed exciton-plasmon and exciton–exciton elementary excitation branches.

Keywords: Magnetoexciton, Bose-Einstein Condensation, Plasmon.

CONTENTS

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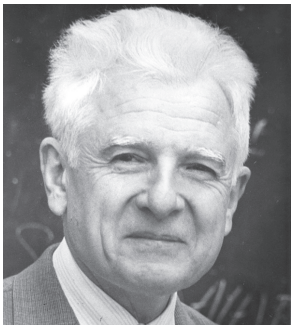

Moskalenko Sveatoslav Anatol was born on September 26, 1928 in the village Bravicha Calarash district, Republic of Moldova. Citizenship of Moldova. Moskalenko Sveatoslav Anatol has graduated the Kishinev State University in 1951. In 1956–1959 he was post-graduate student of the Institute of Physics of the Academy of Sciences of Ukrainian SSR. He became Candidate of Physico-mathematical Sciences equivalent to Ph.D. degree in 1961. Doctor of Physico-mathematical Sciences and the professor in the field of theoretical and mathematical physics beginning with 1971 and 1974 correspondingly. He became Laureate of the State prize in the field of Sciences and Technics of MSSR and USSR in 1981 and 1988 correspondingly. He was elected member-correspondent and full member of the Academy of Sciences of Moldova in 1989 and 1992 correspondingly. The concept of excitonic molecule was introduced. Later the exitonic molecule was called the biexciton. The biexciton represents the bound state of four Fermi quasiparticles (quaternions), namely, two electrons and two holes. More simple it can be regarded as a bound state of two excitons. The possibility of Bose-Einstein condensation (BEC) of quasiparticles with finite lifetime such as excitons and biexcitons was suggested. It was pointed out that BEC can occur in the quasiequilibrium conditions, when the relaxation time due to interparticle scattering is much less than their life time and interaction between excitons is repulsive. The superfluidity of excitons and biexcitons can provide a new way of non-dissipative energy transfer in crystals. BEC can be induced by the resonant monochromatic photons. The results concerning the Bose-Einstein condensation of excitons and biexcitons as well the coherent nonlinear optics with excitons were reviewed in the monograph written together with Professor D. W. Snoke from Pittsburg University. Due to the collaboration with Professor M. A. Liberman from Uppsala University last years the properties of excitons in a strong magnetic field are studied. The polarizability, correlation energy and the dielectric liquid phase of Bose-Einstein condensed 2D magnetoexcitons with motional dipole moments were studied. The possible existence of the metastable dielectric liquid phase formed by Bose-Einstein condensed magnetoexcitons with wave vectors and motional dipole moments different from zero was established theoretically.

Michael (Mikhail) Liberman was born in Moscow, USSR on October 23, 1942. He graduated from Moscow State University in 1966. From 1969 to 2003 he worked at P. Kapitsa Institute for Physical Problems, Academy of Sciences USSR. He received his Ph.D. in 1971 from P. Lebedev Physical Institute in Moscow for the group theory in quantum mechanics and invariant expansion of the relativistic amplitudes, and then his Doctor of Physical and Mathematical Sciences degree in 1981 for a thesis on ionizing shock waves. Since 1991, he is professor of theoretical statistical physics working at the Physics Department, Uppsala University, Sweden. He is a citizen of both Russia and the Sweden. Among his achievements are the nonlinear theory of electromagnetic wave propagating in nonequilibrium plasmas (for example, in the ionosphere); a theory of the ionizing shock waves, exact solution for a hydrogen atom in a magnetic field of arbitrary strength, theory of a hydrogen molecule in a strong magnetic field, non-stationary nonlinear equation for a curved flame, theory of type Ia supernova explosion. At Uppsala University he continue his work on combustion theory, for which he was recently nominated for Gold Medal of Combustion Institute, and he is also focused on the research of the Bose-Einstein condensate of excitons in low dimensional semiconductors in a strong magnetic field. He is author of the books: "Physics of Shock Waves in Gases and Plasmas," Springer-Verlag, 1985 (with A. Velikovich), "Physics of High-Density Z-pinch Plasmas," Springer-Verlag, 1998 (with J. DeGroot, A. Toor, R. Spielman), "Introduction to Physics and Chemistry of Combustion," Springer-Verlag, 2008.

Dumanov Evgheni Victor was born on September 30, 1982 in the Chisinau city, Republic of Moldova. Citizenship of Moldova. E. V. Dumanov has graduated the Moldova State University in 2004. In 2004–2007 he was post-graduate student of the Institute of Applied Physics of the Academy of Sciences of Moldova. He received his Ph.D. in 2008 from Institute of Applied Physics in Chisinau for the theoretical and mathematical physics. His research interests are collective properties of elementary particle in semiconductors, effects of Bose-Einstein condensation 2D magnetoexcitons.

1. INTRODUCTION

In the past two decades a number of experimental[1–4] and theoretical[5–13] efforts have been dedicated to the study of 2D systems in a strong magnetic field. Lerner, Lozovik and Dzynbenko[5–7] firstly have been studied the coherent pairing of electrons and holes resulting in the formation of the Bose–Einstein condensate (BEC-te) of excitons in a single-particle state with wave vector $k = 0$. Even beyond the Hartree–Fock approximation, it was possible to obtain an exact solution if the coupling to higher Landau levels and the corresponding correlation energy can be neglected. In this case the magnetoexcitons at $T = 0$ represent an ideal excitonic gas. In spite of the different from zero Coulomb interaction between the electrons, between the holes and between electrons and holes the resulting interaction between two magnetoexcitons equals exactly to zero. This extraordinary result is due to the fact that the cyclotron orbits of the electron and hole forming exciton with $k = 0$ are exactly superposed one over other and completely coincide having the same radius in spite of the fact that their cyclotron frequencies are different if the electron and hole masses m_e, m_h are different. The radii of the cyclotron orbits equal to $l\sqrt{2}$ where l is the magnetic length and do not depend on the masses m_i, where $i = e, h$. The square magnetic length equals to $l^2 = \hbar c/eH$, where H is the magnetic field strength. It is supposed to be strong, so that the distances between the Landau levels for electrons and for holes $\hbar\omega_{c,i} = \hbar eH/m_i c$ are much bigger than the binding energy of the 2D exciton, and the magnetic length l will be smaller than the radius a_{ex} of 2D exciton. The critical value of the magnetic field which obeys to these conditions is

$$H_{er} = \frac{4cl^3\mu^2}{\hbar^3\varepsilon_0^2}; \quad \mu = \frac{m_e m_h}{m_e + m_h}$$

For typical values of the GaAs crystal $\varepsilon_0 = 12.56$; $\mu \leq m_e = 0.067m_0$; $a_{ex} = 100$ Å we can find $H_{er} = 6.57$ T. Another surprising result is that the droplets of the metallic-type electron–hole liquid (EHL) have the minimal energy per electron–hole (e–h) pair, when the local filling factor ν of the lowest Landau level (LLL) is maximal i.e., $\nu = 1$. The electron–hole droplets (EHD) can be considered as an aggregate of excitons sticked together.[5, 6, 8, 9] The coupling to higher Landau levels makes the system weakly nonideal,[5, 6] which allows the Berezinskii–Kosterlitz–Thouless topological phase transition[14–16] at finite temperature. The results obtained in Refs. [5–7] were reproduced in Ref. [8] on the basis of more simple and transparent approach using the BCS-type wave function of the BEC-ed excitons and calculating the ground state energy in the Hartree–Fock–Bogoliubov approximation (HEBA). But in the addition the authors[8] took into account the indirect interaction between the particles lying on the LLL due to their simultaneous virtual excitation during the Coulomb interaction to higher Landau levels and their return back to the initial states. This indirect interaction taken into account in the frame of HEBA is equivalent to the calculation of the correlation energy of the BEC-ed excitons when their polarizability is conditioned by the excitation of the charged particles from the LLL to higher Landau levels. We will return to this important question below, reconsidering the problem and presenting an effective Hamiltonian and concrete estimations of the underlying physical effects.

The starting wave functions of the electrons and holes on the 2D semiconductors structure in a strong perpendicular magnetic field were determined in the Landau gauge and are characterized by the numbers of the Landau levels which appear due to the Landau quantization in one direction of the plane and by the wave vector of the translational motion in another in-plane direction perpendicular to the previous one. The magnetic field transforms the kinetic energy of the translational motion into the potential energy of the one dimensional oscillator with a shifted equilibrium position, as it is shown in Figure 1. The position on the plane of the point of gyration of electron is determined by its wave vector. There are N possible positions of the gyration point on the plane with surface area S and this number equals to $N = S/2\pi l^2$, where $2\pi l^2$ is the area of the cyclotron orbit. Now it is useful to note that the flux of the magnetic field through the quantum orbit gives rise to the quantum of the magnetic flux ϕ_0 which is determined as

$$\phi_0 = H2\pi l^2 = hc/e; \quad h = 2\pi\hbar$$

The total number of magnetic flux quanta through the area surface S equals exactly to the degeneracy manifold

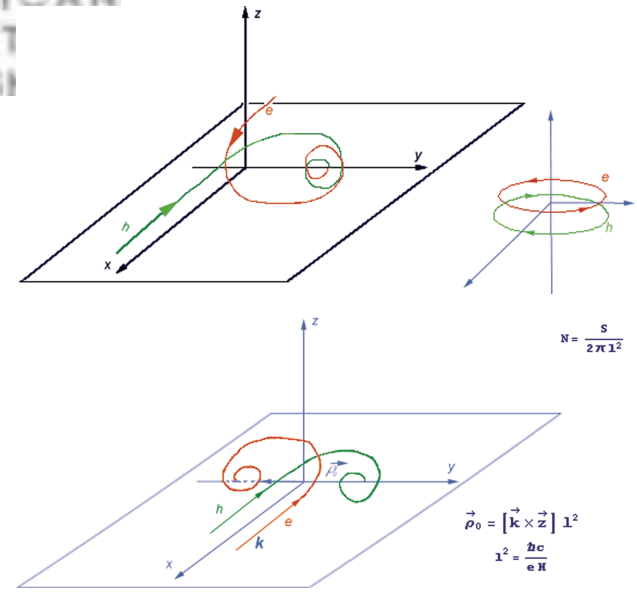

Fig. 1. The motion of the electron–hole pair in a strong perpendicular magnetic field. The action of the Lorentz force on the counter-propagating and co-propagating e–h pairs.

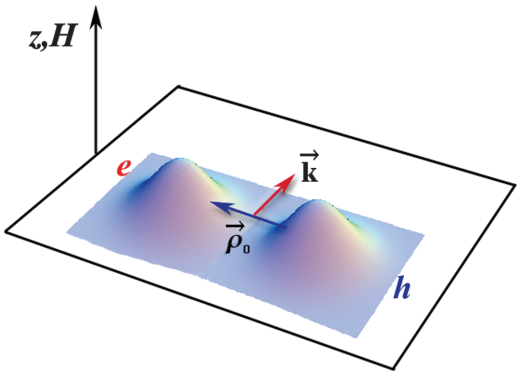

Fig. 2. The electron–hole structure of the magnetoexciton.

N of the given Landau level. This detail will play a crucial role when the states of the spatially separated electrons and holes in the double quantum well structure in the presence of perpendicular electric and magnetic fields will be discussed.

To visualize the behavior of the 2D e–h pair in a strong magnetic field one can look on the Figure 1 where two counter—propagating electron and hole in x direction are shifted by the Lorentz force in the same part of the y axis. Their cyclotron orbits are overposed exactly in this case. If the electron and hole take part together in the exciton center of mass motion, each of them carrying some part of the resultant wave vector $\vec{k} \neq 0$, in this case their centers of gyration and their cyclotron orbits are shifted by the Lorentz force in opposite parts of the y axis.

The distance between the gyration points appears, which determine the motional dipole moment $\vec{\rho}_0$

$$\vec{\rho}_0 = \frac{[\vec{H} \times \vec{k}]}{H} l^2; \quad \rho_0 = kl^2$$

It is an in-plane vector perpendicular to the wave vector of translational motion $\vec{k}$. This case is also represented in the Figure 1. The quantum orbits of the electron and hole around their gyration points have the radii $l\sqrt{2}$ and the area surfaces $2\pi l^2$ indifferent on the value of the motional dipole moment ρ_0.

The radii of the quantum orbits $l\sqrt{2}$ and the distance between them ρ_0 are completely determined by the strong magnetic field and the exciton formation, is due to e–h Coulomb interaction. The electron–hole structure of the magnetoexciton is represented in the Figure 2. Because the distance ρ_0 can be changed continuously by changing the wave vector k, the binding energy of the magnetoexciton also will change continuously giving rise to the continuous energy band of relative e–h motion in dependence on the wave vector k.

2. THE SCREENING EFFECTS AND CORRELATION ENERGY

Nozieres and Comte[18] proposed a method that permits one to take into account simultaneously the binding processes and the screening effects in a system of the Bose condensed e–h pairs. It can be used both in the dense limit $n_{ex}a_{ex}^3 \gg 1$, in which it essentially coincides with the RPA[19] for a two-component plasma, and in the dilute limit $n_{ex}a_{ex}^3 \ll 1$, in which it is able to describe in part the Kramers polarizability of the neutral atomic excitons and the van der Waals interaction between them.

The formulation of the method, called the generalized RPA (GRPA), was accompanied by detailed explanations of the underlying physics and is very instructive. As a main element it contains the coherence factor $(u_p v_q - u_q v_p)^2$, which is typical of a BCS ground state. The main results of the Nozieres and Comte paper[18] concerning the Bose condensed excitons are presented here.

Following the Pauli-Feynman theorem,[19, 20] the ground-state energy E_0 of the system of interacting e–h pairs can be expressed in the form

$$E_0 = E_{kin} + \int_0^{e^2} E_{int}(\lambda) \frac{d\lambda}{\lambda} \tag{1}$$

which contains the kinetic energy E_{kin} of the Bose condensed ideal e–h pairs without the Coulomb interaction between them and the mean value of their Coulomb interaction $E_{int}(\lambda)$ with a variable value of the electric charge squared, λ. The variational parameter λ changes from zero to the value of a real electron charge squared, e^2. To calculate $E_{int}(\lambda)$, one must also take into account the occurrence of BEC.

The hypothetical e–h gas has the wave function $|n(\lambda)\rangle$ that depends on λ and the bare Coulomb interaction,

$$V_k(\lambda) = V_k \frac{\lambda}{e^2}; \quad V_k = \frac{4\pi e^2}{\varepsilon_0 k^2 V} \tag{2}$$

The free-particle energy spectra and their total number do not depend on λ. The Coulomb interaction can be expressed in a factorized form

$$\frac{1}{2} \sum_k V_k \left[\rho_k \rho_k^\dagger - N_e - N_h\right] \tag{3}$$

where the charge density ρ_k and the full electron and hole number operators N_e and N_h are

$$\begin{aligned} \rho_k &= \sum_p a_{p+k}^\dagger a_p - \sum_p b_{p+k}^\dagger b_p \\ N_e &= \sum_p a_p^\dagger a_p; \quad N_h = \sum_p b_p^\dagger b_p \end{aligned} \tag{4}$$

Here the spin-index summation is dropped. The expression $E_{int}(\lambda)$ can be written as

$$E_{int}(\lambda) = -N_{ex} \sum_k V_k(\lambda) + \frac{1}{2} \sum_k V_k(\lambda) \sum_{n(\lambda)} \left|\left(\rho_k^\dagger\right)_{no}\right|^2 \tag{5}$$

The first term subtracts the self-interaction of electrons and holes from the factorized part of the Coulomb interaction. N_{ex} is the mean number of e–h pairs or excitons $N_{ex} = N_e = N_h$.

As noted by Nozieres and Comte, the screening corrections can act on only real electron transitions allowed by the exclusion principle, leaving the self-interaction term untouched.

The matrix elements in Eq. (5) can be expressed in terms of the imaginary part of the dynamical dielectric constant $\varepsilon(k, \omega, \lambda)$ as

$$\frac{1}{2}\sum_k V_k(\lambda)\sum_{n(\lambda)}\left|\left(\rho_k^{\dagger}\right)_{no}\right|^2 = -\int_0^{\infty}\frac{\hbar d\omega}{2\pi}\mathrm{Im}\left[\frac{1}{\varepsilon(k,\omega,\lambda)}\right] \quad (6)$$

Relation (6) has been deduced in the theory of the linear response of the system to a weak perturbation.[19, 20] In as much as the system is characterized by the variable λ, $\varepsilon(k, \omega, \lambda)$ also depends on λ. The ground-state energy of Eq. (1) then obtains the form

$$E_0 = E_{\mathrm{kin}} - N_{\mathrm{ex}}\sum_k V_k - \sum_k\int_0^{\infty}\frac{\hbar d\omega}{2\pi}\int_0^{e^2}\frac{d\lambda}{\lambda}\mathrm{Im}\left[\frac{1}{\varepsilon(k,\omega,\lambda)}\right] \quad (7)$$

The choice of the approximation for $\varepsilon(k, \omega, \lambda)$ determines the accuracy of the determination of the energy E_0. For the one-component plasma and the electron–hole liquid, the best results were obtained with the RPA together with Hubbard-type corrections.[19–21]

In the framework of the RPA, the effective Coulomb interaction between two electrons in a one-component plasma can be represented as a sum of ring diagrams, as in Figure 3. All the wavy lines in Figure 3 have the same transfer wave vector k and the same contributions V_k. The exchange Coulomb diagrams are neglected, which are characterized by the appearance of the contributions V_{k_F} instead of V_k, where k_F is the Fermi wave vector. In the dense limit, this approximation is valid because the actual values of k are much less than those of k_F and $V_{k_F} \ll V_k$. But in the intermediate metallic range, $n_{\mathrm{ex}}a_{\mathrm{ex}}^3 \approx 1$, the exchange diagrams must be taken into account when k exceeds k_F. In this case, the direct and the exchange Coulomb interactions of two electrons with the same spin projections cancel each other, and the Hubbard corrections to the usual RPA are needed. Summing up the diagrams in Figure 3, one obtains the effective Coulomb interaction $V_{\mathrm{eff}}(k, \omega)$ and the dynamical dielectric constant $\varepsilon^{\mathrm{RPA}}(k, \omega)$ as follows

$$V_{\mathrm{eff}}(k,\omega) = \frac{V_k}{\varepsilon^{\mathrm{RPA}}(k,\omega)} = V_k + V_k\prod(k,\omega)V_k + \cdots$$
$$\varepsilon^{\mathrm{RPA}}(k,\omega) = 1 - V_k\prod(k,\omega) = 1 + 4\pi\alpha(k,\omega) \quad (8)$$

Here $\alpha(k, \omega)$ is the polarizability of the electron plasma, whereas $\prod(k, \omega)$ is the polarization-loop contribution, which can be expressed through the free-electron Green's functions $G_e^o(p, \omega')$ in the form

$$\prod(k,\omega) = -i\sum_q\int_{-\infty}^{\infty}\frac{\hbar d\omega'}{2\pi}G_e^o(q,\omega')G_e^o(q+k,\omega+\omega') \quad (9)$$

In the case of Bose condensed e–h pairs, as a zeroth-order Hamiltonian one must choose the expression $\tilde{H}_0$ in Eq. (10).

$$\tilde{H}_0 = \frac{1}{2}\sum_p E(P)\left(\alpha_p^{\dagger}\alpha_p + \beta_p^{\dagger}\beta_p\right) \quad (10)$$

where $E(P)$ is the energy needed for the excitation of one free e–h pair from the exciton Bose condensate. It is diagonalized in the new quasiparticle operators α_p and β_p, which are connected to the operators a_p and b_p of the initial electrons and holes by relations:

$$a_p = u_p\alpha_p - v_p\beta_p^{\dagger}$$
$$b_p = u_p\beta_p + v_p\alpha_p^{\dagger} \quad (11)$$

In virtue of this, there are four zeroth-order Green's functions constructed from the operators $a_p^{\dagger}$, a_p, $b_p^{\dagger}$ and b_p as follows:

$$G_a^0(p,t) = -i\langle T[a_p^{\dagger}(t)a_p(0)]\rangle_0 \quad \overset{a}{\longrightarrow}\ \overset{a}{\longrightarrow}$$
$$G_b^0(p,t) = -i\langle T[b_p^{\dagger}(t)b_p(0)]\rangle_0 \quad \overset{b}{\longrightarrow}\ \overset{b}{\longrightarrow}$$
$$\hat{G}^0(p,t) = -i\langle T[a_p^{\dagger}(t)b_{-p}^{\dagger}(0)]\rangle_0 \quad \overset{a}{\longrightarrow}\ \overset{b}{\longleftarrow} \quad (12)$$
$$\breve{G}^0(p,t) = -i\langle T[b_{-p}(t)a_p(0)]\rangle_0 \quad \overset{b}{\longleftarrow}\ \overset{a}{\longrightarrow}$$

Here one must calculate the time dependence of the operators a_p and b_p in the interaction representation and the averaging by using the Hamiltonian $\tilde{H}_0$ in Eq. (10) before going from operators a_p and b_p to the new operators α_p and β_p. The Fourier transforms of the Green's function (12) for the case $m_e = m_h$, which is considered here, are

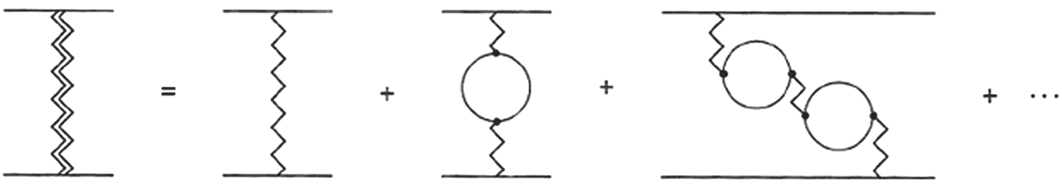

Fig. 3. The sum diagram containing the polarization loops in the RPA. Reprinted with permission from [19], D. Pines, Elementary Excitations in Solids, Benjamin, New York **(1963)**. © 1963, W. A. Benjamin.

$$G_a^0(p,\omega) = G_b^0(p,\omega) = \left[\frac{u_p^2}{\hbar\omega + E(p)/2 - i\delta} + \frac{v_p^2}{\hbar\omega - E(p)/2 + i\delta}\right] \tag{13}$$

$$\hat{G}^0(p,\omega) = \breve{G}(p,\omega) = u_p v_p\left[\frac{1}{\hbar\omega + E(p)/2 - i\delta} - \frac{1}{\hbar\omega - E(p)/2 + i\delta}\right]$$

In this case, the role of the simple polarization loop in Figure 3 is played by a compound polarization loop, the contribution of which consists of three terms,

$$\Pi(k,\omega) = \Pi_a(k,\omega) + \Pi_b(k,\omega) + 2\Pi_{ab}(k,\omega) \tag{14}$$

They correspond to the three simple polarization loops represented in Figure 4. Two of them are determined by the Green's functions $G_a^0(q,\omega')$ and $G_b^0(q,\omega')$, whereas the third one is determined by the anomalous functions $\hat{G}^0(q,\omega')$ and $\breve{G}^0(q,\omega')$ as follows:

$$\Pi_a(k,\omega) = \Pi_b(k,\omega) = -i\sum_q \int_{-\infty}^{\infty} \frac{\hbar d\omega'}{2\pi} G_a^0(q,\omega') G_a^0(q+k,\omega'+\omega) \tag{15}$$

$$\Pi_{ab}(k,\omega) = -i\sum_q \int_{-\infty}^{\infty} \frac{\hbar d\omega'}{2\pi} \hat{G}^0(q,\omega') \breve{G}^0(q+k,\omega'+\omega)$$

As was determined by Nozieres and Comte, the summed polarization contribution is

$$\Pi(k,\omega) = -\sum_q \left(u_{k+q}v_q - u_q v_{k+q}\right)^2 \times \left[\frac{1}{\hbar\omega + (E(k+q)+E(q))/2 - i\delta} - \frac{1}{\hbar\omega - (E(k+q)+E(q))/2 + i\delta}\right] \tag{16}$$

This determined the polarizability of the *e–h* system

$$4\pi\alpha(k,\omega) = -V_k\Pi(k,\omega) \tag{17}$$

After separation of the real and the imaginary parts of the polarizability, one obtains the dispersional $\varepsilon_1(k,\omega)$ and the absorptional $\varepsilon_2(k,\omega)$ components of the dynamical dielectric constant,

$$\varepsilon(k,\omega) = \varepsilon_1(k,\omega) + i\varepsilon_2(k,\omega)$$

$$\varepsilon_1(k,\omega) = 1 + 4\pi\alpha_1(k,\omega) = 1 - V_k\Pi_1(k,\omega) = 1 + V_k\sum_q \frac{(u_{k+q}v_q - u_q v_{k+q})^2 (E(k+q)+E(q))}{((E(k+q)+E(q))/2)^2 - \hbar^2\omega^2}$$

$$\varepsilon_2(k,\omega) = 4\pi\alpha_2(k,\omega) = -V_k\Pi_2(k,\omega) = \pi V_k\sum_q (u_{k+q}v_q - u_q v_{k+q})^2 \times \left[\delta\left(\hbar\omega - \frac{E(k+q)+E(q)}{2}\right) + \delta\left(\hbar\omega + \frac{E(k+q)+E(q)}{2}\right)\right] \tag{18}$$

The coherence factor $(u_{k+q}v_q - u_q v_{k+q})^2$ is typical of a BCS ground state. One can verify that for a steplike v_q, the polarizability $\alpha(k,\omega)$ reduces to the sum $\alpha_e(k,\omega) + \alpha_h(k,\omega)$ of normal electron and hole polarizabilities.

To determine the ground-state energy of the Bose condensed *e–h* pairs, Nozieres and Comte substituted into main formula (6) the expression for $\varepsilon(k,\omega,\lambda)$ found in the framework of the GRPA, of the form

$$\varepsilon(k,\omega,\lambda) = 1 - V_k(\lambda)\Pi(k,\omega) \tag{19}$$

where the explicit dependence on λ is due to the bare Coulomb interaction constant $V_k(\lambda)$. When Eq. (19) is used, the following formula can be obtained:

$$\int_0^{e^2} \frac{d\lambda}{\lambda} \mathrm{Im}\left[\frac{1}{\varepsilon(k,\omega,\lambda)}\right] = \arctan\frac{V_k\Pi_2(k,\omega)}{1 - V_k\Pi_1(k,\omega)} \tag{20}$$

If one assumes that $|\varepsilon_2(k,\omega)/\varepsilon_1(k,\omega)| < 1$ and substitutes $\arctan x$ by x, formula (7) takes the form

$$E_0 = E_{\mathrm{kin}} - N_{\mathrm{ex}}\sum_k V_k - \sum_k V_k \int_0^{\infty} \frac{\hbar d\omega}{2\pi} \frac{\Pi_2(k,\omega)}{1 - V_k\Pi_1(k,\omega)} \tag{21}$$

To obtain the ground-state energy in the HFBA, it is enough to neglect the polarizability $V_k(\lambda)\Pi_1(k,\omega)$ compared with unity in the denominator of formula (21). Then E_{HFB} is

$$E_{\mathrm{HFB}} = E_{\mathrm{kin}} - N_{\mathrm{ex}}\sum_k V_k + \frac{1}{2}\sum_k V_k \sum_q \left(u_{k+q}v_q - u_q v_{k+q}\right)^2 \tag{22}$$

One can represent the ground-state energy as a sum of two terms. One of them is determined in the HFBA and the second, additional part is called a correlation energy, E_{corr}, where $E_0 = E_{\mathrm{HFB}} + E_{\mathrm{corr}}$.

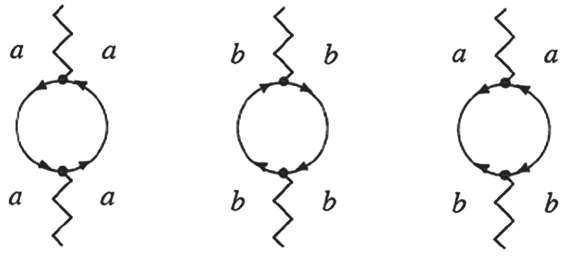

Fig. 4. Three polarization loops in the case of Bose condensed *e–h* pairs. Reprinted with permission from [18], P. Nozieres and C. Comte, *J. Phys.* 43, 1083 (**1982**). © 1982, IOP.

3. POLARIZABILITY, CORRELATION ENERGY, AND DIELECTRIC LIQUID PHASE OF BOSE-EINSTEIN CONDENSATE OF TWO-DIMENSIONAL EXCITONS IN A STRONG PERPENDICULAR MAGNETIC FIELD

The numbers of electrons and holes which occupy the lowest Landau levels are determined by the experimental conditions, e.g., the generation rate due to laser excitation and the exciton recombination rate. Assuming an quasipermanent population with an equal number of electrons and holes, their quasiequilibrium states are characterized by the chemical potentials μ_e and μ_h. The energy of electrons and holes as well as their chemical potentials are measured relative to their lowest Landau levels. The exciton formation reaction $e+h \leftrightarrow ex$ implies the relation between the chemical potentials $\mu_e+\mu_h=\mu_{ex}$

The coherent macroscopic state corresponding to Bose-Einstein condensation of correlated electron–hole pairs in a single-particle state with the wave vector k can be introduced following Keldysh–Kozlov–Kopaev method (KKK method)[22, 23] by applying the unitary transformation[8, 23, 24]

$$D(\sqrt{N_{\rm ex}}) = \exp\left[\sqrt{N_{\rm ex}}(d_k^\dagger - d_k)\right] = \prod_t \exp\left[\sqrt{2\pi l^2 n_{\rm ex}}(e^{-ik_y tl^2} a^\dagger_{k_x/2+t} b^\dagger_{k_x/2-t} - e^{ik_y tl^2} b_{k_x/2-t} a_{k_x/2+t})\right] \quad (23)$$

where

$$N_{\rm ex}/N = 2\pi l^2 n_{\rm ex}, \quad n_{\rm ex} = N_{\rm ex}/S \quad (24)$$

Following the Refs. [23, 24] we introduced the BCS-type wave function of the new coherent macroscopic state acting on the electron–hole vacuum state $|0\rangle$ by the unitary transformation operator $D(\sqrt{N_{\rm ex}})$

$$|\psi_g(k)\rangle = D\left(\sqrt{N_{\rm ex}}\right)|0\rangle = \prod_t (u + ve^{-ik_y tl^2} a^\dagger_{k_x/2+t} b^\dagger_{k_x/2-t})|0\rangle \quad (25)$$

where the coefficients u and v are

$$u = \mathrm{Cos}g, \quad v = \mathrm{Sin}g, \quad g = \sqrt{2\pi l^2 n_{\rm ex}}, \quad u^2+v^2=1 \quad (26)$$

The transformed Hamiltonian is

$$\tilde{H} = D\left(\sqrt{N_{\rm ex}}\right) H D^\dagger\left(\sqrt{N_{\rm ex}}\right) \quad (27)$$

For this Hamiltonian, the new ground state wave function plays the same role as the initial vacuum state $|0\rangle$ for the Hamiltonian H,

$$\tilde{H}|\psi_g(k)\rangle = DHD^\dagger D|0\rangle = DH|0\rangle = 0 \quad (28)$$

The unitary transformation of the Hamiltonian H means the unitary transformations of the operators a_p, b_p

$$Da_pD^\dagger = \alpha_p = ua_p - v(p-k_x/2)\, b^\dagger_{k_x/2-p}$$
$$Db_pD^\dagger = \beta_p = ub_p + v(k_x/2-p)\, a^\dagger_{k_x/2-p} \quad (29)$$

where

$$v(t) = ve^{-ik_y tl^2}$$
$$v(t)v(s) = vv(t+s) \quad (30)$$
$$v^*(t) = v(-t)$$

and the inverse transformation is

$$a_p = u\alpha_p + v(p-k_x/2)\,\beta^\dagger_{k_x-p}$$
$$b_p = u\beta_p - v(k_x/2-p)\,\alpha^\dagger_{k_x-p} \quad (31)$$

The average numbers of the electrons and holes in the new ground state at $T=0$ can be determined from equalities

$$\langle\psi_g(k)|\, a_p^\dagger a_p\, |\psi_g(k)\rangle = \langle\psi_g(k)|\, b_p^\dagger b_p\, |\psi_g(k)\rangle = \nu^2 \quad (32)$$

This expression means that the total average numbers of electrons, holes and excitons are

$$N_{\rm ex} = \sum_p \langle\psi_g(k)|\, a_p^\dagger a_p\, |\psi_g(k)\rangle = N\nu^2, \quad n_{\rm ex} = \nu^2/2\pi l^2 \quad (33)$$

The applicability of the theory is therefore restricted to values of the filling factor v^2 defined by $v^2 \approx \mathrm{Sin}v^2$. This restriction reflects the fact the excitons are the compound particles composed from two fermions. Their creation and annihilation operators can be considered as Bose operators only at small concentrations of the electron–hole pairs. Below we shall assume $v^2 \le 1/4$.

Following the KKK method, the transformed Hamiltonian $\tilde{H} = DHD^\dagger$ must be expressed in terms of the new operators α_p, $a_p^\dagger$, β_p and $\beta_p^\dagger$ using Bogoliubov's u, v transformations. In this way the Hamiltonian $\tilde{H}$ can be represented in the form

$$\tilde{H} = U + H_2 + H' \quad (34)$$

The first term U does not contain operators α_p and β_p and plays the role of the new ground state energy. The second term H_2 is quadratic in the operators α_p and β_p and appears as a result of transpositions of the new operators and their normal ordering. In this transposition, the commutation relations of the Fermi operators α_p and β_p transform terms with four operators into quadratic terms. The term H' contains the remaining normal-ordered terms with four operators, which is treated as a perturbation. The term U can be represented as

$$U = N_{\rm ex}[E_{\rm ex}(k)-\mu] - N_{\rm ex}v^2[I_l - I_{\rm ex}(k)] = -N_{\rm ex}[I_{\rm ex}(k)+\mu] - N_{\rm ex}v^2[I_l - I_{\rm ex}(k)] \quad (35)$$

where

$$N_{\rm ex} = Nv^2, \quad E_{\rm ex}(k) = -I_{\rm ex}(k),$$
$$I_{\rm ex}(k) = I_l e^{-k^2l^2/4} I_0\left(k^2l^2/4\right) \quad (36)$$

The last term containing the correction proportional to filling factor v^2, is negative due to the inequality $I_{\rm ex}(k) \le I_l$. The term H_2 contains diagonal quadratic terms as well as the terms describing the creation and annihilation of the new e–h pairs from the new vacuum state $|\psi_g(k)\rangle$. It has the form

$$H_2 = \sum_p E(k, v^2, \mu)(a_p^\dagger a_p + b_p^\dagger b_p) - \sum_p [uv(k_x/2 - p)\psi(k, v^2, \mu)\beta_{k_x-p}\alpha_p + uv(p - k_x/2)\psi(k, v^2, \mu)\alpha_p^\dagger \beta_{k_x-p}^\dagger] \quad (37)$$

where

$$E(k, v^2, \mu) = 2u^2v^2 I_{\rm ex}(k) + I_l(v^4 - u^2v^2) - \mu(u^2 - v^2)/2 \quad (38)$$

and

$$\psi(k, v^2, \mu) = 2v^2 I_l + I_{\rm ex}(k)(u^2 - v^2) - \mu \quad (39)$$

It is seen from the Hamiltonian H_2 that the new quasiparticles described by the operators α_p and β_p can appear spontaneously from the new vacuum state as a pair with total momentum k_x, which coincides with the translational wave vector of the Bose-Einstein condensate of magnetoexcitons. Such terms in the Hamiltonian and the corresponding diagrams are called dangerous ones and make the new vacuum state unstable. To avoid this instability, the condition of compensation of the dangerous diagrams is used. In the Hartree–Fock–Bogoliubov approximation, when only the dangerous diagrams in H_2 are taken into account, the condition of their compensation is

$$\psi(k, v^2, \mu) = 0 \quad (40)$$

This condition determines the unknown parameter of the theory, namely the chemical potential μ of the system. In the Hartree–Fock–Bogoliubov approximation it is

$$\mu^{\rm HFB} = E_{\rm ex}(k) - 2v^2[I_l - I_{\rm ex}(k)] = -I_{\rm ex}(k) - 2v^2[I_l - I_{\rm ex}(k)] \quad (41)$$

This condition introduces the breaking of the $u(1)$ gauge symmetry of the initial Hamiltonian H and makes nonequivalent its ground state and the new ground state U. With the help of $\mu^{\rm HFB}$ we can determine self-consistently the ground state energy U and the energy of the single-particle elementary excitations, which in the given approximation are

$$U^{\rm HFB} = N_{\rm ex} v^2[I_l - I_{\rm ex}(k)], \quad E(k, v^2, \mu) = \frac{1}{2}E_{\rm ex}(k) \quad (42)$$

As one can see, the single-particle elementary excitation has an energy spectrum without dispersion. It does not depend on the wave vectors p or $k_x - p$ of the electron or of the hole appearing from the new vacuum state, and for each particle is equal exactly to one half of the ionization energy of the condensed excitons. To excite one electron–hole pair from the vacuum, the energy $I_{\rm ex}(k)$ is required, because it is equivalent to an unbound single exciton with the wave vector k. The absence of dispersion reflects the absence of the kinetic energy of the electrons and holes in the lowest Landau level. It was shown Refs. [8, 24] that in the frame of the linearized equations of the motion for the electron and hole density operators there are no plasma oscillations in the case $k = 0$, whereas the dispersion relation of the collective excitations in this case is given by the exciton dispersion relation $E_{\rm ex}(k) - E_{\rm ex}(0)$. These results will be generalized when condensation of excitons with dipole moments occurs. In this case, the ground state energy can be determined with BCS-type wave functions by the expression

$$E_g(k) = \langle\psi_g(k)|H|\psi_g(k)\rangle = -Nv^2 I_{\rm ex}(k) - Nv^4[I_l - I_{\rm ex}(k)] \quad (43)$$

It was obtained in the frame of the Hartree–Fock approximation. Its derivative $dE_g(k)/dN_{\rm ex}$ determines the chemical potential $\mu^{\rm HFB}$ in full accordance with Eq. (41), whereas the rate $E_g(k)/N_{\rm ex}$ characterizes the mean energy per one exciton

$$\frac{E_g(k)}{N_{\rm ex}} = E_{\rm ex}(k) - v^2[E_{\rm ex}(k) - E_{\rm ex}(0)] = E_{\rm ex}(k) - v^2[I_l - I_{\rm ex}(k)], \quad E_g(k) = U + \mu^{\rm HFB} N_{\rm ex} \quad (44)$$

Below we show that this result can be extended beyond the Hartree–Fock–Bogoliubov approximation, taking into consideration the polarizability of the Bose-Einstein condensed magnetoexcitons, at least for a symmetric 2D model.

The coherent excited states of the Bose-condensed magnetoexcitons can be constructed following the method proposed by Anderson in the theory of superconductivity.[25] The excited state can be obtained by acting with the electron part of the density fluctuation operator on the ground state wave function,

$$|\psi^e(q \pm Q_x/2)\rangle = a^\dagger_{q+Q_x/2} a_{q-Q_x/2}|\psi_g(k)\rangle \quad (45)$$

Using the operators α_p and β_p, the u–v transformation, and taking into account

$$\alpha_p|\psi_g(k)\rangle = \beta_p|\psi_g(k)\rangle = 0 \quad (46)$$

this function can be reduced to

$$|\psi^e(q \pm Q_x/2)\rangle = [vv(-Q_x)\beta_{k_x-q-Q_x/2}\beta^\dagger_{k_x-q+Q_x/2} + uv(q - k_x/2 - Q_x/2) \times \alpha^\dagger_{q+Q_x/2}\beta^\dagger_{k_x-q-Q_x/2}]|\psi_g(k)\rangle \quad (47)$$

Taking into account that Q_x is different from zero, one can simplify last equation neglecting the first term in the right hand side. The set of functions obeys the following orthogonality and normalization conditions

$$\langle\psi^e(p\pm P_x/2)|\psi^e(q\pm Q_x/2)\rangle = u^2v^2\delta_{kr}(P_x,Q_x)\delta_{kr}(p,q) \tag{48}$$

The excitation energy in the Hartree–Fock–Bogoliubov approximation can be found using the H_2 part of the Hamiltonian

$$H_2 = \sum_p \frac{I_{\rm ex}(k)}{2}(\alpha_p^\dagger\alpha_p + \beta_p^\dagger\beta_p) \tag{49}$$

and equals

$$E(q\pm Q_x/2) = \frac{\langle\psi^e(q\pm Q_x/2)|H_2|\psi^e(q\pm Q_x/2)\rangle}{\langle\psi^e(q\pm Q_x/2)|\psi^e(q\pm Q_x/2)\rangle} = I_{\rm ex}(k) \tag{50}$$

The excitation energy of this state equals the ionization potential of Bose-condensed magnetoexcitons with wave vector k. It does not depend on the wave vectors q and Q_x, which characterize the excited state. The excitation energy and the energy spectrum of single-particle elementary excitations are the same for the full set of the excited states and have no dispersion.

We also introduce the excited states generated by the fluctuation of the hole density and by the action of the corresponding operator on the ground state wave function $|\psi_g(k)\rangle$

$$\begin{aligned}&\left|\psi^h(-p\pm P_x/2)\right\rangle \\ &= \left[vv(P_x)\alpha_{k_x+p-P_x/2}\alpha^\dagger_{k_x+p+P_x/2} + uv(p+k_x/2 + P_x/2)\alpha^\dagger_{k_x+p+P_x/2}\beta^\dagger_{-p+P_x/2}\right]|\psi_g(k)\rangle\end{aligned} \tag{51}$$

In a similar way, the excited wave functions generated by the pair of electron and hole creation operators are introduced in the form

$$\left|\psi^{e-h}\left(\pm p, \frac{P_x+k_x}{2}\right)\right\rangle = a^\dagger_{p+(P_x+k_x)/2}b^\dagger_{-p+(P_x+k_x)/2}|\psi_g(k)\rangle \tag{52}$$

The matrix elements of the density operators can be calculated using these excited-state wave functions

$$|(\rho_Q)_{n,0}|^2 = \left|\frac{1}{uv}\langle\psi^e(p\pm P_x/2)|\rho_Q|\psi_g(k)\rangle\right|^2 \tag{53}$$

and they determine the polarizability of the system. In our case

$$\rho_Q|\psi_g(k)\rangle = \sum_q e^{iQ_yql^2}\left(|\psi^e(q\mp Q_x/2)\rangle - |\psi^h(-q\mp Q_x/2)\rangle\right) \tag{54}$$

We shall calculate the polarizability in the approximation of a weak response to the external longitudinal perturbation.[19,26] In the case of a 2D structure with Hamiltonian of the form (3), the perturbation caused by an external probe charge can be written in the form

$$H_{\rm ext}(t) = -\frac{ez}{e^2}W_Q\left[\rho_{\rm ext}(Q,\omega)\rho_Q^\dagger e^{-i\omega t} + \rho^*_{\rm ext}(Q,\omega)\rho_Q e^{i\omega t}\right] \tag{55}$$

The specific properties of a 2D structure appear due to the Coulomb interaction coefficient W_Q, which differs essentially from the case of 3D structure. Substituting (53) into a general expression for the polarizability, we find

$$4\pi\alpha_0^{\rm HF}(Q,\omega) = -\frac{W_Q}{\hbar}\sum_n\left[\frac{|(\rho_Q^\dagger)_{n,0}|^2}{\omega-\omega_{n,0}+i\delta} - \frac{|(\rho_Q)_{n,0}|^2}{\omega+\omega_{n,0}+i\delta}\right] \tag{56}$$

This expression can be rewritten as

$$\begin{aligned}&4\pi\alpha_0^{\rm HF}(Q,\omega) \\ &\underset{\delta\to+0}{=} -4u^2v^2\frac{e^2}{\varepsilon_0 l^2|Q|}e^{-Q^2l^2/2}\sin^2\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right) \\ &\quad\times\left[\frac{1}{\hbar\omega-I_{\rm ex}(k)+i\delta} - \frac{1}{\hbar\omega+I_{\rm ex}(k)+i\delta}\right]\end{aligned} \tag{57}$$

The same result for the polarizability can be obtained from another possible set of excitations. The polarizability has a resonance frequency equal to the ionization potential $I_{\rm ex}(k)$ of the magnetoexciton with dipole moment $\rho_0 = kl^2$. The polarizability vanishes when the wave vector k approaches zero, and the magnetoexcitons behave as an ideal noninteracting gas. The polarizability is an anisotropic function on the wave vector Q and decreases exponentially when Q goes to infinity.

The real and imaginary parts of the polarizability are

$$\begin{aligned}4\pi\alpha_0^{\rm HF}(Q,\omega) &= 4\pi\alpha_{0,1}^{\rm HF}(Q,\omega) + i4\pi\alpha_{0,2}^{\rm HF}(Q,\omega) \\ 4\pi\alpha_{0,1}^{\rm HF}(Q,\omega) &= -4u^2v^2(W_QN)\sin^2\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right) \\ &\quad\times\left[\frac{Pf}{\hbar\omega-I_{\rm ex}(k)} - \frac{Pf}{\hbar\omega+I_{\rm ex}(k)}\right] \\ 4\pi\alpha_{0,2}^{\rm HF}(Q,\omega) &= 4\pi u^2v^2(W_QN)\sin^2\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right) \\ &\quad\times\{\delta[\hbar\omega-I_{\rm ex}(k)] - \delta[\hbar\omega+I_{\rm ex}(k)]\}\end{aligned} \tag{58}$$

The symbol (Pf) denotes that the singular term which may appear in the expression $4\pi\alpha_{0,1}^{\rm HF}(Q,\omega)$ at the point $\hbar\omega = I_{\rm ex}(k)$ must be removed. The polarizability $4\pi\alpha_0^{\rm HF}(Q,\omega)$ determines the dielectric constant $\varepsilon(Q,\omega)$, which in the Hartree–Fock (HF) approximation is

$$\frac{1}{\varepsilon^{\rm HF}(Q,\omega)} = 1 - 4\pi\alpha_0^{\rm HF}(Q,\omega) \tag{59}$$

contrary to the random phase approximation (RPA), where the expression for the dielectric constant is different.

The dielectric constant $\varepsilon(Q,\omega)$ obtained by this method is denoted $\varepsilon^{\mathrm{RPA}}(Q,\omega)$:

$$\varepsilon^{\mathrm{RPA}}(Q,\omega)=1+4\pi\alpha_0^{\mathrm{HF}}(Q,\omega) \tag{60}$$

Now we calculate the correlation energy of the condensed excitons, taking into account the screening effects. As was mentioned in the previous section, for this purpose it is convenient to use the method proposed in Ref. [18], which allows us consider simultaneously the binding processes and the screening effects in a system of Bose condensed *e–h* pairs, which can be used both in the cases of dense and dilute limits of exciton concentrations. The formulation of this method, called the generalized random-phase approximation (GRPA), is based on the Pauli-Feynman theorem[17–19] for the ground state energy. In the case of interacting 2D *e–h* pairs, their ground state energy E_0 also can be expressed in the form (1).

Here E_{kin} is the kinetic energy of the Bose condensed ideal *e–h* pairs without Coulomb interaction between them, and $E_{\mathrm{int}}(\lambda)$ is the mean value of the Coulomb interaction, with λ being the square electric charge, which changes from zero to the real value e^2. For the gas of electrons and holes occupying the lowest Landau levels only the Coulomb interaction term enters the Hamiltonian. Applying the GRPA method, we introduce the hypothetical gas with the bare Coulomb interaction $W_Q(\lambda)=W_Q(\lambda/e^2)$ and wave functions $|n(\lambda)\rangle$. Then the interaction part of the ground state energy is

$$W_Q=\int_0^{e^2}\frac{W_Q(\lambda)}{\lambda}d\lambda \tag{61}$$

The expression $E_{\mathrm{int}}(\lambda)$ takes the form

$$E_{\mathrm{int}}(\lambda)=-N_{\mathrm{ex}}\sum_Q W_Q(\lambda)+\frac{1}{2}\sum_Q W_Q(\lambda)\sum_{n(\lambda)}|(\rho_Q^{\dagger})_{n,0}|^2 \tag{62}$$

which can be expressed through the imaginary part of the polarizability $4\pi\alpha_0^{\mathrm{HF}}(Q,\omega)$ obtained in the Hartree-Fock approximation.

For the ground state energy we find

$$E_0=-N_{\mathrm{ex}}\sum_Q W_Q-\sum_Q\int_0^{\infty}\frac{\hbar d\omega}{2\pi}\int_0^{e^2}\frac{d\lambda}{\lambda}\mathrm{Im}\frac{1}{\varepsilon(Q,\omega,\lambda)} \tag{63}$$

Representing $\varepsilon(Q,\omega,\lambda)$ in the form

$$\begin{aligned}\varepsilon(Q,\omega,\lambda)&=\varepsilon_1(Q,\omega,\lambda)+i\varepsilon_2(Q,\omega,\lambda)\\ \varepsilon_1(Q,\omega,\lambda)&=1+4\pi\alpha_{0,1}^{\mathrm{HF}}(Q,\omega,\lambda)\\ \varepsilon_2(Q,\omega,\lambda)&=4\pi\alpha_{0,2}^{\mathrm{HF}}(Q,\omega,\lambda)\end{aligned} \tag{64}$$

and supposing $\varepsilon_2(Q,\omega,\lambda)\approx\lambda$ and $\varepsilon_1(Q,\omega,\lambda)\approx 1$, we obtain

$$\begin{aligned}\int_0^{e^2}\frac{d\lambda}{\lambda}\mathrm{Im}\frac{1}{\varepsilon(Q,\omega,\lambda)}&=-\frac{1}{\varepsilon_1(Q,\omega)}\arctan\frac{\varepsilon_2(Q,\omega)}{\varepsilon_1(Q,\omega)}\\ &\approx\frac{\varepsilon_2(Q,\omega)}{\varepsilon_1^2(Q,\omega)}\end{aligned} \tag{65}$$

Substituting Eq. (64) into this expression and expanding it in series up to the first order in $4\pi\alpha_{0,1}^{\mathrm{HF}}(Q,\omega)$, inclusively we obtain

$$\begin{aligned}\frac{4\pi\alpha_{0,2}^{\mathrm{HF}}(Q,\omega)}{\left[1+4\pi\alpha_{0,1}^{\mathrm{HF}}(Q,\omega)\right]}&\approx 4\pi\alpha_{0,2}^{\mathrm{HF}}(Q,\omega)-2\times 4\pi\alpha_{0,2}^{\mathrm{HF}}(Q,\omega)\\ &\times 4\pi\alpha_{0,1}^{\mathrm{HF}}(Q,\omega)\end{aligned} \tag{66}$$

The first term in the series expansion corresponds to Hartree–Fock–Bogoliubov approximation, whereas the second term determines the correlation energy

$$E_{\mathrm{corr}}=-2\sum_Q\int_0^{\infty}\frac{\hbar d\omega}{2\pi}4\pi\alpha_{0,2}^{\mathrm{HF}}(Q,\omega)4\pi\alpha_{0,1}^{\mathrm{HF}}(Q,\omega) \tag{67}$$

As explained above, only the real part of the polarizability $4\pi\alpha_{0,1}^{\mathrm{HF}}(Q,\omega)$, which contains the denominator $\hbar\omega+I_{\mathrm{ex}}(k)$, gives a nonvanishing contribution to the correlation energy at the point $\hbar\omega=I_{\mathrm{ex}}(k)$. At this point the denominator equals $2I_{\mathrm{ex}}(k)$, which means that the correlation energy is due to the virtual excitation of two quasiparticle pairs out of the mean-field ground state $|\psi_g(k)\rangle$. The correlation energy is negative due to the screening effects and therefore lowers the energy of the interacting system. However, this reduction is not monotonic and the local minimum of the correlation energy depends on the value of the filling factor v^2.

We obtain the correlation energy in the form

$$\begin{aligned}E_{\mathrm{corr}}&=-\frac{8\left(u^2v^2\right)^2}{I_{\mathrm{ex}}(k)}\sum_Q\left(W_QN\right)^2\mathrm{Sin}^4\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right)\\ &=-\frac{N\left(u^2v^2\right)^2}{\sqrt{\pi}}\frac{I_l^2}{I_{\mathrm{ex}}(k)}F(kl)\end{aligned} \tag{68}$$

$F(kl)$ can be presented through the modified Bessel function $I_0(z)$ as follows

$$F(kl)=3+e^{-k^2l^2/2}I_0\left(k^2l^2/2\right)-4e^{-k^2l^2/8}I_0\left(k^2l^2/8\right) \tag{69}$$

The total mean energy per *e–h* pair equals

$$\varepsilon=\varepsilon^{\mathrm{HF}}+\varepsilon_{\mathrm{corr}}=\frac{E_g(k)+E_{\mathrm{corr}}}{N_{\mathrm{ex}}} \tag{70}$$

The corresponding correction to the chemical potential is

$$\begin{aligned}\mu_{\mathrm{corr}}&=\frac{dE_{\mathrm{corr}}}{dN_{\mathrm{ex}}}=\frac{d}{dv^2}\frac{E_{\mathrm{corr}}}{N}\\ N_{\mathrm{ex}}&=Nv^2\end{aligned} \tag{71}$$

and the total value of the chemical potential is

$$\mu = \mu^{\mathrm{HFB}} + \mu_{\mathrm{corr}} = -I_{\mathrm{ex}}(k) - 2v^2[I_l + I_{\mathrm{ex}}(k)] - \frac{2}{\sqrt{\pi}} \frac{I_l^2 F(kl)}{I_{\mathrm{ex}}(k)} v^2(1-v^2)(1-2v^2) \qquad (72)$$

The mean energy per particle and the chemical potential versus the filling factor v^2 are shown in Figures 5 and 6. The first term in Eq. (72) gives the energy per exciton in the Hartree–Fock–Bogoliubov approximation. For small values of kl, the total energy and chemical potential, with the correlation corrections, are monotonic functions of v^2, and almost coincide with those found in the frame of the HFBA. For larger kl, the total energy and chemical potential deviate considerably from their values in the HFBA and become nonmonotonic functions of v^2 with a well-pronounced local minimum. This minimum becomes deeper and more pronounced with the increase of the dipole moment kl^2, due to the increase of the coherence factor and the decrease of the ionization potential $I_{\mathrm{ex}}(k)$.

At first, the local minimum of chemical potential appears for $kl = 2.2$; it becomes deeper with further increase of kl. The relative minimum of the chemical potential of the Bose-Einstein condensed magnetoexcitons implies the formation of a metastable dielectric liquid phase with positive compressibility in this range of filling factor v^2. At the values $kl = 4.6$, $I_{\mathrm{ex}}(k) = 0.18 I_l$, $\rho_0 = 4.6l$, and $v^2 = 0.25$, the minimum on the plot of the chemical potential achieves the same value as at the limiting point $v^2 = 1$. In spite of the fact that the curves in Figures 5 and 6 are extrapolated up to the point $v^2 = 1$, it should be noticed that the applicability of the theory is limited by the upper boundary given by $v^2 \approx \mathrm{Sin}^2 v$.

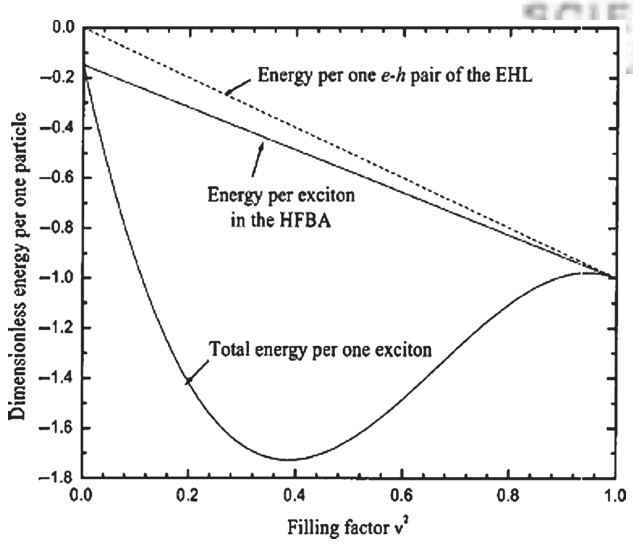

Fig. 5. Energy per e–h pair versus filling factor v^2. Solid line: energy per exciton at $kl = 4.6$. Dashed line: energy per one exciton in the Hartree–Fock–Bogoliubov approximation. Dash-dotted line: energy per e–h pair of the metallic EHL.

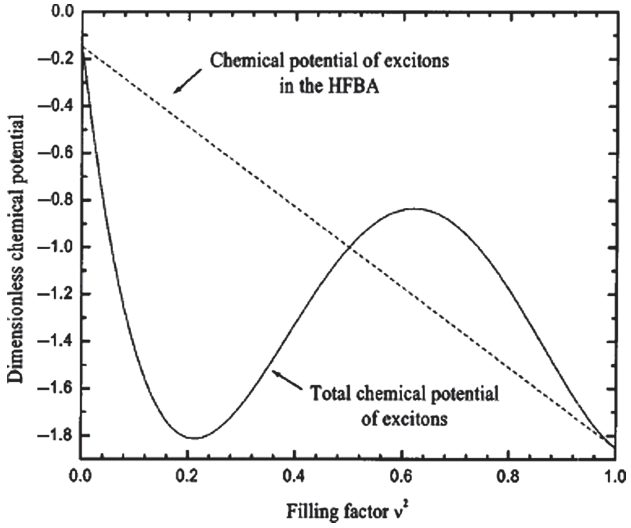

Fig. 6. Chemical potential versus filling factor v^2. Solid line: total chemical potential of the condensed magnetoexcitons at $kl = 4.6$. Dashed line: chemical potential of the condensed magnetoexcitons in HFBA at $kl = 4.6$.

4. COLLECTIVE ELEMENTARY EXCITATIONS OF TWO-DIMENSIONAL MAGNETOEXCITONS IN THE BOSE-EINSTEIN CONDENSATION STATE WITH k DIFFERENT FROM ZERO

The creation and annihilation operators of magnetoexcitons are two-particle operators reflecting the electron–hole (e–h) structure of the excitons. They are denoted below as $d^+(\vec{p})$ and $d(\vec{p})$, where $\vec{p}(p_x, p_y)$ is the two-dimensional wave vector. There are also the density fluctuation operators for electrons $\hat{\rho}_e(\vec{Q})$ and for holes $\hat{\rho}_h(\vec{Q})$ as well as their linear combinations $\hat{\rho}(\vec{Q})$ and $\hat{D}(\vec{Q})$. They are determined below

$$\begin{aligned} \hat{\rho}_e(\vec{Q}) &= \sum_t e^{iQ_y t l^2} a^\dagger_{t-(Q_x/2)} a_{t+(Q_x/2)} \\ \hat{\rho}_h(\vec{Q}) &= \sum_t e^{iQ_y t l^2} b^\dagger_{t+(Q_x/2)} b_{t-(Q_x/2)} \\ \hat{\rho}(\vec{Q}) &= \hat{\rho}_e(\vec{Q}) - \hat{\rho}_h(-\vec{Q}) \\ \hat{D}(\vec{Q}) &= \hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q}) \\ d^+(\vec{P}) &= \frac{1}{\sqrt{N}} \sum_t e^{-iP_y t l^2} a^\dagger_{t+(P_x/2)} b^\dagger_{-t+(P_x/2)} \\ d(\vec{P}) &= \frac{1}{\sqrt{N}} \sum_t e^{iP_y t l^2} b_{-t+(P_x/2)} a_{t+(P_x/2)} \\ \hat{N}_e &= \hat{\rho}_e(0); \quad \hat{N}_h = \hat{\rho}_h(0) \\ \hat{\rho}(0) &= \hat{N}_e - \hat{N}_h; \quad \hat{D}(0) = \hat{N}_e + \hat{N}_h \end{aligned} \qquad (73)$$

and are expressed through the Fermi creation and annihilation operators $a_p^\dagger$, a_p for electrons and $b_p^\dagger$, b_p for holes. The e–h Fermi operators depend on two quantum numbers. In Landau gauge one of them is the wave number p and the second one is the quantum number n of the Landau level. In the lowest Landau level (LLL) approximation n has only the value zero and its notation is dropped. The wave number p enumerates the N-fold degenerate states of the 2D electrons in a strong magnetic field. N can be expressed through the layer surface area S and the magnetic length l as follows: $N=S/(2\pi l^2)$; $l^2=(\hbar c)/(eH)$, where H is the magnetic field strength. The operators (73) obey to the following commutation relations, most of which were discussed for the first time in the papers[29,30]

$$
\begin{aligned}
&[\hat\rho(\vec Q),\hat\rho(\vec P)]=-2i\mathrm{Sin}\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)\hat\rho(\vec P+\vec Q)\\
&[\hat D(\vec Q),\hat D(\vec P)]=-2i\mathrm{Sin}\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)\hat\rho(\vec P+\vec Q)\\
&[\hat\rho(\vec Q),\hat D(\vec P)]=-2i\mathrm{Sin}\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)\hat D(\vec P+\vec Q)
\end{aligned}
\tag{74}
$$

$$
\begin{aligned}
[d(p),d^+(Q)] &= \delta_{kr}(\vec P,\vec Q)\\
&-\frac{1}{N}\left[i\mathrm{Sin}\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)\hat\rho(\vec P-\vec Q)\right.\\
&\left.+\mathrm{Cos}\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)\hat D(\vec P-\vec Q)\right]
\end{aligned}
$$

This list can be continued

$$
\begin{aligned}
&[\hat\rho(\vec P),d(\vec Q)]=2i\mathrm{Sin}\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)d(\vec P+\vec Q)\\
&[\hat\rho(\vec P),d^+(\vec Q)]=-2i\mathrm{Sin}\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)d^+(-\vec P+\vec Q)\\
&[\hat D(\vec P),d^+(\vec Q)]=2\mathrm{Cos}\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)d^+(\vec Q-\vec P)\\
&[\hat D(\vec P),d(\vec Q)]=-2\mathrm{Cos}\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)d(\vec P+\vec Q)
\end{aligned}
\tag{75}
$$

One can observe that the density fluctuation operators (73) with different wave vectors $\vec P$ and $\vec Q$ do not commute. Their non-commutativity is related with the vorticity which accompanies the presence of the strong magnetic field and depends on the vector-product of two wave vectors $\vec P$ and $\vec Q$ and its projection on the direction of the magnetic field $[\vec P\times\vec Q]_z$. These properties considerably influence on the structure of the equations of motion for the operators (73) and determine new aspect of the magneto-exciton-plasmon physics. Indeed in the case of 3D e–h plasma in the absence of the external magnetic field the density fluctuation operators do commute.[19] The magneto-exciton creation and annihilation operators $d^\dagger(\vec p)$ and $d(\vec Q)$ as in general case do not obey exactly to the Bose commutation rule. Their deviation from it is proportional to the density fluctuation operators $\hat\rho(\vec P-\vec Q)$ and $\hat D(\vec P-\vec Q)$. The discussed above operators determine the structure of the 2D e–h system Hamiltonian in the LLL approximation.

The starting Hamiltonian $\hat{\mathscr{H}}$ in the quasi average theory approximation (QATA)[31] has the form

$$
\begin{aligned}
\hat{\mathscr{H}} &= \frac{1}{2}\sum_{\vec Q} W_{\vec Q}[\rho(\vec Q)\rho(-\vec Q)-\hat N_e-\hat N_h]-\mu_e\hat N_e-\mu_h\hat N_h\\
&-\eta\sqrt{N}(e^{i\varphi}d^\dagger(k)+e^{-i\varphi}d(k))
\end{aligned}
\tag{76}
$$

The equations of motion for the operators (73) are obtained using the commutation relations (74), (75). They are

$$
\begin{aligned}
i\hbar\frac{d}{dt}d(\vec P) &= [d(\vec P),\hat{\mathscr{H}}]\\
&= (E(\vec P)-\bar\mu)d(\vec P)-\eta\sqrt{N}e^{i\varphi}\delta_{kr}(\vec P,\vec K)\\
&-2i\sum_{\vec Q}W_{\vec Q}\mathrm{Sin}\left(\frac{[\vec P\times\vec Q]_z l^2}{2}\right)\hat\rho(\vec Q)d(\vec P-\vec Q)\\
&+\eta e^{i\varphi}\left[i\mathrm{Sin}\left(\frac{[\vec P\times\vec K]_z l^2}{2}\right)\frac{\hat\rho(\vec P-\vec K)}{\sqrt{N}}\right.\\
&\left.+\mathrm{Cos}\left(\frac{[\vec P\times\vec K]_z l^2}{2}\right)\frac{\hat D(\vec P-\vec K)}{\sqrt{N}}\right]
\end{aligned}
\tag{77}
$$

$$
\begin{aligned}
&i\hbar\frac{d}{dt}d^\dagger(2\vec K-\vec P)\\
&=[d^\dagger(2\vec K-\vec P),\hat{\mathscr{H}}]\\
&=(\bar\mu-E(2\vec K-\vec P))d^\dagger(2\vec K-\vec P)+\eta\sqrt{N}e^{-i\varphi}\delta_{kr}(\vec P,\vec K)\\
&-2i\sum_{\vec Q}W_{\vec Q}\mathrm{Sin}\left(\frac{[(2\vec K-\vec P)\times\vec Q]_z l^2}{2}\right)\\
&\times d^\dagger(2\vec K-\vec P-\vec Q)\hat\rho(-\vec Q)\\
&-\eta e^{-i\varphi}\left[i\mathrm{Sin}\left(\frac{[\vec P\times\vec K]_z l^2}{2}\right)\frac{\hat\rho(\vec P-\vec K)}{\sqrt{N}}\right.\\
&\left.+\mathrm{Cos}\left(\frac{[\vec P\times\vec K]_z l^2}{2}\right)\frac{\hat D(\vec P-\vec K)}{\sqrt{N}}\right]
\end{aligned}
$$

$$
\begin{aligned}
&i\hbar\frac{d}{dt}\hat\rho(\vec P-\vec K)\\
&=[\hat\rho(\vec P-\vec K),\hat{\mathscr{H}}]\\
&=-i\sum_{\vec Q}W_{\vec Q}\mathrm{Sin}\left(\frac{[(\vec P-\vec K)\times\vec Q]_z l^2}{2}\right)\\
&\times[\hat\rho(\vec P-\vec K-\vec Q)\hat\rho(\vec Q)+\hat\rho(\vec Q)\hat\rho(\vec P-\vec K-\vec Q)]\\
&-2i\eta\sqrt{N}\mathrm{Sin}\left(\frac{[\vec P\times\vec K]_z l^2}{2}\right)[e^{-i\varphi}d(\vec P)-e^{i\varphi}d^\dagger(2\vec K-\vec P)]
\end{aligned}
$$

$$
\begin{aligned}
&i\hbar\frac{d}{dt}\hat{D}(\vec{P}-\vec{K})\\
&=[\hat{D}(\vec{P}-\vec{K}),\hat{\mathscr{H}}]\\
&=-i\sum_{\vec{Q}}W_{\vec{Q}}\mathrm{Sin}\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right)\\
&\times[\hat{\rho}(\vec{Q})\hat{D}(\vec{P}-\vec{K}-\vec{Q})+\hat{D}(\vec{P}-\vec{K}-\vec{Q})\hat{\rho}(\vec{Q})]\\
&+2\eta\sqrt{N}\mathrm{Cos}\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right)\\
&\times[e^{-i\varphi}d(\vec{P})-e^{i\varphi}d^{\dagger}(2\vec{K}-\vec{P})]
\end{aligned}
$$

Here

$$\eta=(E_{\mathrm{ex}}(K)-\mu)v=(E(K)-\overline{\mu})v;\quad v=v^2;\quad N_{\mathrm{ex}}=v^2N \tag{78}$$

Now we must pay attention to one important aspect of these equations of motion, closely related with the non-commutativity of the operators (73) expressed by the formulas (74) and (75). Applying them one can prove the equivalent expressions for the exciton operator $d(P)$

$$
\begin{aligned}
&(E(P)-\bar{\mu})d(P)-2i\sum_{\vec{Q}}W_{\vec{Q}}\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\rho(\vec{Q})d(\vec{P}-\vec{Q})\\
&=-\bar{\mu}d(P)-i\sum_{\vec{Q}}W_{\vec{Q}}\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)[\rho(\vec{Q})d(\vec{P}-\vec{Q})\\
&+d(\vec{P}-\vec{Q})\rho(\vec{Q})]=-(\bar{\mu}+E(P))d(P)\\
&-2i\sum_{\vec{Q}}W_{\vec{Q}}\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d(\vec{P}-\vec{Q})\rho(\vec{Q})=\cdots
\end{aligned}
\tag{79}
$$

as well as for the density fluctuation operator $\hat{\rho}(P)$

$$
\begin{aligned}
&-i\sum_{\vec{Q}}W_{\vec{Q}}\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)[\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})+\hat{\rho}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})]\\
&=E(P)\hat{\rho}(P)-2i\sum_{\vec{Q}}W_{\vec{Q}}\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})\\
&=-E(P)\hat{\rho}(P)-2i\sum_{\vec{Q}}W_{\vec{Q}}\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\\
&\times\hat{\rho}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})=\cdots
\end{aligned}
\tag{80}
$$

They can be verified taking into account the relation

$$2\sum_{\vec{Q}}W_{\vec{Q}}\mathrm{Sin}^2\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)=E(P) \tag{81}$$

The quantum of the Coulomb energy $E(P)$ is related with the helicity and vorticity existing in the frame of electron–hole (e–h) system in the presence of a strong perpendicular magnetic field. A full spectrum of these quanta with arbitrary wave vector $\vec{P}$ does exist not only in the case of e–h system, but also in the case of pure electron or pure hole systems. We are supposing that their origin is related with the existence of N magnetic flux quanta $\phi_0=hc/e$ discussed in Ref. [32] in the case of FQHE. The flux quanta enforce the creation of N vortices in the 2DEG, lead to the creation of composite fermions and bosons, accompanying the transport phenomena and so on.[32] The unlimited reservoir of the Coulomb interaction energy between electrons in the presence of the magnetic field is characterized by energy quanta $E(P)$, which depend only on the square electric charge e^2 and magnetic length l and does not depend on the e–h densities. They could be named as Coulomb magnetic energy quanta. In our previous paper[33] they were named as plasmon quanta, but on our opinion it is better to conserve the name of plasmon quanta to the intra-Landau level excitations whose energy depend on the filling factors.

As follows from the equalities (80), (81) the induced by vortices the Coulomb magnetic energy quanta can be added or substracted as a free part terms outside the nonlinear terms, if we will change simultaneously the corresponding nonlinear terms.

In the case of matter interacting with the resonant laser radiation with the frequency ω_L in the rotating reference frame the energy of quasiparticles is changed by the photon quantum energy $\hbar\omega_L$. Such type of energy which appears in the case of unlimited reservoir of energy is named as quasi-energy.[34] The new supplementary quasi-energy branches give rise to many effects gathered by the common mane as optical Stark effect.[17] On our opinion something similar takes place in the presence of a strong magnetic field, but in difference on the laser radiation with a well defined frequency $\omega_L=ck_0$ and wave vector k_0, in the case of a strong magnetic field there are a large spectra of frequencies and wave vectors. Adding or extracting the quanta $E(P)$ we can form many virtual complexes of quasiparticles with different free energies. They can be named as quasi-energy complexes. As we will see below the most of them will have great damping rates and will be physical meaningless. The choosing of the concrete forms of equations of motion depends in great manner on the theoretical methods, which we intend to apply.

We will apply below the Green's function method. In this case the free energy terms in the equation of motion for operators as usual play the role of the proper energies in zero order approximation. They can determine the zero-order Green's function, whereas the nonlinear terms can be taken into account in higher order of the perturbation theory. Of course, when the equations of motion for the Green's function are treated exactly in this case it is indifferent which starting variant was selected, because all of them are completely equivalent. But in reality it is impossible to solve exactly the infinite chains of equations of

motion for Green's functions and some concrete approximations are inevitable.

Taking into account these considerations we have chosen the equations of motion for the exciton creation and annihilation operators $d^{\dagger}(P), d(P)$ with a free energy term accounted from the exciton chemical potential in the form $(E_{\rm ex}(P)-\mu)$. $E_{\rm ex}(P)$ coincides with the energy of the magnetoexciton without any corrections depending on the exciton–exciton interaction, what means without concentration corrections. The equations of motion for the density fluctuation operators $\rho(\vec{P})$ and $D(\vec{P})$ were chosen in the first variant of the Eq. (80) without free energy terms, because the proper energies of the intra-lowest Landau level excitations depend on the filling factors and can not be represented by quanta $E(P)$ in any forms. The true expressions for the plasmon eigenenergies will appear in the second order of the perturbation theory developed on concentration parameter, and its value will depend on $v^2(1-v^2)$. Another important consideration for the selection of the starting equations of motion having in view the Green's function method, is the damping rates of the obtained elementary excitations. The imaginary parts of the eigen-energies of the elementary excitations depend on the real Coulomb scattering processes with the participations of the quasiparticles as well as on their free energies which appear in zero order approximation. Below we will show that in most cases the damping rates are of the same order of magnitude as the corresponding real parts due the absence of small parameter related with Coulomb energy. It means that such elementary excitations can not exist and have not any physical meaning.

Once again we can underline that it happens because the Coulomb interaction energy can not be considered as a small perturbation. In fact there is a unique possibility to chose the equations of motion for the operators $\rho(\vec{P})$ and $D(\vec{P})$ as it was realized in our equations of motion. One can represent different variants of equations of motion with different free energy terms as corresponding to different quasienergy complexes consisting from quasiparticles and Coulomb magnetic quanta $E(P)$. This suggestion is supported and induced by the well known concept of composite particles created by electrons and magnetic flux quanta ϕ_0[32] and by the supposition that their existence must be evidenced also in another phenomena not so far from the FQHE. But trying to do it, and verifying the consequences posteriori we arrived to the conclusion that most of them have great damping rates and do not exist. The unique possibility to obtain in the frame of the Green's function method the intra-LLL excitations of the plasmon type without damping at all is the variant chosen by us and written above in the frame of the equations of motion (77). But for the magnetoexcitons some different quasienergy complexes are possible. Here we will discuss only the variant with an usual dispersion law. On the base of equations of motion (77) the Green's functions will be introduced and the chains of equations of motion for the Green's functions will be developed.

Following the equations of motion (77) we will introduce four interconnected retarded Green's functions at $T=0$[35,36]

$$
\begin{aligned}
G_{11}(\vec{P},t) &= \langle\langle d(\vec{P},t)d^{\dagger}(\vec{P},0)\rangle\rangle \\
G_{12}(\vec{P},t) &= \langle\langle d^{\dagger}(2\vec{K}-\vec{P},t);d^{\dagger}(\vec{P},0)\rangle\rangle \\
G_{13}(\vec{P},t) &= \left\langle\left\langle \frac{\hat{\rho}(\vec{P}-\vec{K},t)}{\sqrt{N}};d^{\dagger}(\vec{P},0)\right\rangle\right\rangle \\
G_{14}(\vec{P},t) &= \left\langle\left\langle \frac{\hat{D}(\vec{P}-\vec{K},t)}{\sqrt{N}};d^{\dagger}(\vec{P},0)\right\rangle\right\rangle
\end{aligned}
\tag{82}
$$

They are determined by the relations

$$
\begin{aligned}
G(t) &= \langle\langle \hat{A}(t);\hat{B}(0)\rangle\rangle = -i\theta(t)\langle[A(t),B(0)]\rangle \\
\hat{A}(t) &= e^{(iHt)/\hbar}\hat{A}e^{-(iHt)/\hbar} \\
[\hat{A},\hat{B}] &= \hat{A}\hat{B}-\hat{B}\hat{A}
\end{aligned}
\tag{83}
$$

Where $\hat{H}$ is the Hamiltonian (76).

The average $\langle\ \rangle$ will be calculated at $T=0$ in HFB approximation using the ground state wave function $|\psi_g(k)\rangle$ (25). The time derivative of the Green's function is calculated as follows

$$
\begin{aligned}
i\hbar\frac{d}{dt}G(t) &= i\hbar\frac{d}{dt}\langle\langle A(t);B(0)\rangle\rangle \\
&= \hbar\delta(t)\langle[\hat{A}(0),\hat{B}(0)]\rangle + \left\langle\left\langle i\hbar\frac{d}{dt}A(t);B(0)\right\rangle\right\rangle \\
&= \hbar\delta(t)C + \langle\langle[\hat{A}(t),\hat{H}];\hat{B}(0)\rangle\rangle
\end{aligned}
\tag{84}
$$

By C will be denoted the average values, which do not depend on t. They are not needed in an explicit form for the determination of the energy spectrum of the elementary excitations.

Fourier transforms of the Green's functions (82) will be denoted as

$$
\begin{aligned}
G_{11}(\vec{P},\omega) &= \langle\langle d(\vec{P})|d^{\dagger}(\vec{P})\rangle\rangle_{\omega} \\
G_{12}(\vec{P},\omega) &= \langle\langle d^{\dagger}(2\vec{K}-\vec{P})|d^{\dagger}(\vec{P})\rangle\rangle_{\omega} \\
G_{13}(\vec{P},\omega) &= \left\langle\left\langle \frac{\hat{\rho}(\vec{P}-\vec{K})}{\sqrt{N}}\middle| d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega} \\
G_{14}(\vec{P},\omega) &= \left\langle\left\langle \frac{\hat{D}(\vec{P}-\vec{K})}{\sqrt{N}}\middle| d^{\dagger}(\vec{P})\right\rangle\right\rangle_{\omega}
\end{aligned}
\tag{85}
$$

The two representations are related each-other

$$
G(\vec{P},\omega) = \int_{-\infty}^{\infty} e^{i\omega t}G(\vec{P},t)dt = \int_{0}^{\infty} e^{i\omega t-\delta t}G(\vec{P},t)dt
$$

where the infinitesimal value $\delta \to +0$ guarantees for the retarded Green's function $G(\vec{P},t)$ the convergence of the integral in the interval $(0,\infty)$.

The motion equation in the frequency representation can be deduced on the base of Eq. (84)

$$\begin{aligned}\int_{-\infty}^{\infty} dt e^{i\omega t} i\hbar \frac{dG(t)}{dt} &= i\hbar \int_{0}^{\infty} dt e^{i\omega t-\delta t} \frac{dG(t)}{dt} \\ &= -i\hbar \int_{0}^{\infty} dt G(t) \frac{de^{i\omega t-\delta t}}{dt} \\ &= (\hbar\omega + i\delta) G(\omega) \\ &= C + \int_{-\infty}^{\infty} dt \langle\langle [\hat{A}(t), \hat{H}]; \hat{B}(0)\rangle\rangle e^{i\omega t} \quad (86)\end{aligned}$$

The Green's functions (85) will be named as one-operator Green's functions because they contain in the left hand side of the vertical line only one summary operator of the types $d(P)$, $d^{\dagger}(P)$, $\hat{\rho}(P)$ and $\hat{D}(P)$. At the same time these Green's functions are two-particle Green's functions, because the summary operators (73) are expressed through the products of two Fermi operators. In this sense the Green's functions (85) are equivalent with the two-particle Green's functions introduced by Keldysh and Kozlov in their fundamental paper,[22] forming the base of the theory of high density excitons in the electron–hole description. But in difference on Ref. [22] we are using the summary operators (74), which represent integrals on the wave vectors of relative motions.

The equations of motion for the Green's function (85) are the following

$$\begin{aligned}&[\hbar\omega + \bar{\mu} - E(\vec{P}) + i\delta] G_{11}(\vec{P},\omega) \\ &= C - 2i \sum_{\vec{Q}} W_{\vec{Q}} \mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right) \langle\langle \hat{\rho}(\vec{Q}) d(\vec{P}-\vec{Q}) | d^{\dagger}(\vec{P})\rangle\rangle_{\omega} \\ &\quad + \eta e^{i\varphi} \left[i\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right) G_{13}(\vec{P},\omega)' \right. \\ &\qquad \left. + \mathrm{Cos}\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right) G_{14}(\vec{P},\omega) \right] \quad (87) \\ &[\hbar\omega - \bar{\mu} + E(2\vec{K}-\vec{P}) + i\delta] G_{12}(\vec{P},\omega) \\ &= C - 2i \sum_{\vec{Q}} W_{\vec{Q}} \mathrm{Sin}\left(\frac{[(2\vec{K}-\vec{P})\times\vec{Q}]_z l^2}{2}\right) \\ &\quad \times \langle\langle d^{\dagger}(2\vec{K}-\vec{P}-\vec{Q}) \hat{\rho}(-\vec{Q}) | d^{\dagger}(\vec{P})\rangle\rangle_{\omega} \\ &\quad - \eta e^{-i\varphi} \left[i\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right) G_{13}(\vec{P},\omega) \right. \\ &\qquad \left. + \mathrm{Cos}\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right) G_{14}(\vec{P},\omega) \right]\end{aligned}$$

$$\begin{aligned}&[\hbar\omega + i\delta] G_{13}(\vec{P},\omega) \\ &= C - i \sum_{\vec{Q}} W_{\vec{Q}} \mathrm{Sin}\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right) \\ &\quad \times \left\langle\left\langle \left[\frac{\hat{\rho}(\vec{P}-\vec{K}\vec{Q})}{\sqrt{N}} \hat{\rho}(\vec{Q}) + \hat{\rho}(\vec{Q}) \frac{\hat{\rho}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}} \right] \middle| d^{\dagger}(\vec{P}) \right\rangle\right\rangle_{\omega} \\ &\quad - 2i\eta \mathrm{Sin}\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right) [e^{-i\varphi} G_{11}(\vec{P},\omega) - e^{i\varphi} G_{12}(\vec{P},\omega)] \\ &[\hbar\omega + i\delta] G_{14}(\vec{P},\omega) \\ &= C - i \sum_{\vec{Q}} W_{\vec{Q}} \mathrm{Sin}\left(\frac{[(\vec{P}-\vec{K})\times\vec{Q}]_z l^2}{2}\right) \\ &\quad \times \left\langle\left\langle \left[\hat{\rho}(\vec{Q}) \frac{\hat{D}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}} \right.\right.\right. \\ &\qquad \left.\left.\left. + \frac{\hat{D}(\vec{P}-\vec{K}-\vec{Q})}{\sqrt{N}} \hat{\rho}(\vec{Q}) \right] \middle| d^{\dagger}(\vec{P}) \right\rangle\right\rangle_{\omega} \\ &\quad + 2\eta \mathrm{Cos}\left(\frac{[\vec{P}\times\vec{K}]_z l^2}{2}\right) [e^{-i\varphi} G_{11}(\vec{P},\omega) - e^{i\varphi} G_{12}(\vec{P},\omega)]\end{aligned}$$

The equation of motion (87) for one-operator Green's functions $G_{1j}(\vec{P},\omega)$, where $j=1,2,3,4$, give rise to new two-operator (four-particle) Green's functions of the types $\langle\langle \hat{\rho}(\vec{Q}) d(\vec{P}-\vec{Q}) | d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$, $\langle\langle d^{\dagger}(2\vec{K}-\vec{P}-\vec{Q}) \hat{\rho}(-\vec{Q}) | d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$, $\langle\langle (\hat{\rho}(\vec{P}-\vec{K}-\vec{Q}))/\sqrt{N} \hat{\rho}(\vec{Q}) | d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$ and $\langle\langle (\hat{D}(\vec{P}-\vec{K}-\vec{Q}))/\sqrt{N} \hat{\rho}(\vec{Q}) | d^{\dagger}(\vec{P})\rangle\rangle_{\omega}$ generated by the nonlinear terms in the equations of motion (77) for the operators (73). It is a well known situation described by Zubarev[36] in his review article. For these two-operator Green's functions of the first generation following the rule (86) the new motion equations were deduced. This second step in the frame of the given method will form the second link of an infinite chain of motion equations. Both links constructed in such a way will be exact in the frame of the Hamiltonian (76). These new motion equations will contain in their components new types of three-operator Green's functions of the first generation as well as new types of the two-operator Green's functions of the second generation, and so on.

The truncation procedure was successfully applied in the case of electron–phonon interaction not only for the metals in normal states, but also for the superconductors.

It can be applied also in the case of Bose-Einstein condensed magnetoexcitons. This phenomenon was taken into account for the very beginning by the Bogoliubov method of quasiaverages. The calculations of the average values of the products of two operators exctracted from the left-hand side of the three-operator Green's functions will be made using the ground state wave function of the Bose-Einstein condensed magnetoexcitons. On this base some

supplementary simplifications of the cumbersome expressions will be proposed.

After the truncations and linearizations the multi-operator Green's functions are expressed through the one-operator Green's function $G_{1j}(\vec{P},\omega)$, with $j=1,2,3,4$, and their four equations of motion can be written in a close form introducing the self-energy parts $\Sigma_{ij}(\vec{P},\omega)$ as follows

$$\sum_{j=1}^{4} G_{1j}(\vec{P},\omega)\Sigma_{jk}(\vec{P},\omega)=C_{1k}; \quad k=1,2,3,4 \tag{88}$$

The most of the self-energy parts $\Sigma_{ij}(\vec{P},\omega)$ contain the average values of the two-operator products. They were calculated using the ground state wave function $|\psi_g(k)\rangle$ (25).

Taking into account the exceptional role played by the average value $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ it can be represented

$$\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle=4u^2v^2N\mathrm{Sin}^2\left(\frac{[\vec{K}\times\vec{Q}]_z l^2}{2}\right) \tag{89}$$

In spite of the made approximations concerning the many operator Green's functions and the averages of the two-operator products the obtained self-energy parts remain cumbersome. But there is one possibility to radically simplify the further calculations. It is related with the collinear geometry of the experimental observation of the elementary excitations, when their propagation direction coincide or is exactly opposite with the condensate wave vector $\vec{k}$. This geometry will be discussed in the next section.

The cumbersome dispersion equation is expressed in general form by the determinant equation

$$\det|\Sigma_{ij}(\vec{P},\omega)|=0; \quad \vec{P}=\vec{K}+\vec{q} \tag{90}$$

It can be essentially simplified in collinear geometry, when the wave vectors $\vec{P}$ of the elementary excitations are parallel or antiparallel to the Bose-Einstein condensate wave vector $\vec{k}$. We will represent the wave vectors $\vec{P}$ in the form $\vec{P}=\vec{k}+\vec{q}$, accounting them from the condensate wave vector $\vec{k}$. The relative wave vector $\vec{q}$ will be also collinear to $\vec{k}$. In this case the projections of the wave vector products $[\vec{P}\times\vec{K}]_z$ as well as all coefficients proportional to $\mathrm{Sin}(([\vec{P}\times\vec{K}]_z l^2)/2)$ and a half of the matrix elements $\Sigma_{ij}(\vec{P},\omega)$ in the Eq. (90) vanish. The determinant Eq. (90) disintegrates in two independent equations. One of them concerns only to optical plasmons and has a simple form

$$\Sigma_{33}(\vec{K}+\vec{q};\omega)=0; \quad [\vec{q}\times\vec{K}]_z=0 \tag{91}$$

whereas the second equation contains only the diagonal self-energy parts Σ_{11}, Σ_{22}, Σ_{44} and the quasi-average constant η

$$\Sigma_{11}(\vec{K}+\vec{q},\omega)\Sigma_{22}(\vec{K}+\vec{q},\omega)\Sigma_{44}(\vec{K}+\vec{q},\omega) -2\eta^2(\Sigma_{11}(\vec{K}+\vec{q},\omega)+\Sigma_{22}(\vec{K}+\vec{q},\omega))=0 \tag{92}$$

It determines three interconnected branches. Two of them describe the proper collective excitations of Bose-Einstein condensed magnetoexcitons and the third branch concerns the acoustical plasmons. In spite of the collinear condition $[\vec{q}\times\vec{K}]_z=0$, the Eqs. (91) and (92) and their energy spectra $\omega(\vec{q})$ are not invariant under the inversion operation when $\vec{q}$ is substituted by $-\vec{q}$, because in the system does exist a well defined direction selected by the wave vector $\vec{k}$. By this reason the elementary excitations with wave vector $\vec{q}$ and $-\vec{q}$ have different energies.

The solutions of the dispersion Eq. (92) will be discussed in two limiting cases. One of them is the point $k=0$, where the system behaves as an ideal Bose gas when the excited Landau levels are neglected and another case of considerable values of wave vectors $kl\sim 3-4$, when the Bose-Einstein condensed 2D magnetoexcitons can exist in a form of metastable dielectric liquid phase or of dielectric droplets. But in all cases the average value $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ and other similar expressions are determined in HFBA by the formulas (89). They are characterized by a coherence factor $\mathrm{Sin}^2(([\vec{k}\times\vec{Q}]_z l^2)/2)$, which vanishes in the point $k=0$. All contributions to the self-energy parts proportional to square of Coulomb interaction matrix elements W_Q^2 multiplied by the averages $\langle\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\rangle$ vanish also making a 2D magnetoexciton system a pure ideal gas, when the influence of the excited Landau levels is neglected. This unusual result was revealed for the first time by Lerner and Lozovik[5–7] and was confirmed by Paquet et al.[8] In the case $k=0$ because the vanishing of the averages (90) the self-energy parts become

$$\begin{aligned}\sigma_{11}(\vec{P},\omega)&=\hbar\omega+\bar{\mu}-E(P)\\ \sigma_{22}(\vec{P},\omega)&=\hbar\omega-\bar{\mu}+E(-P)\\ \sigma_{44}(\vec{P},\omega)&=\hbar\omega\end{aligned} \tag{93}$$

and the excitonic part of the dispersion relation as well the acoustic plasmon frequency look as

$$\begin{aligned}\hbar\omega_{\mathrm{ex}}(P)&=\pm\sqrt{(\bar{\mu}-E(P))^2+4\eta^2}\\ \hbar\omega_A(P)&=0\end{aligned} \tag{94}$$

The values $\bar{\mu}=E(k)(1-2v^2)$ and $\eta=(E(k)-\bar{\mu})v=2E(k)v^3$ in the point $k=0$ turn to vanish, i.e., $\bar{\mu}=\eta=E(0)=0$, what leads to the free magnetoexciton dispersion law $\hbar\omega_{\mathrm{ex}}(P)=\pm E(P)$, and coincides with the result obtained earlier in Ref. [8]. The acoustical plasmon branch as well as the optical branch have frequencies equal to zero. The case $k\neq 0$, but $v=0$, can be obtained from the previous formula because, as earlier, the averages (89) as well as the parameter η are vanishing, whereas the chemical potential is different from zero i.e., $\bar{\mu}=E(k)$.

In this case the exciton dispersion law in collinear geometry with $P=k+q\mathrm{Cos}\alpha$ has the form

$$\hbar\omega_{\mathrm{ex}}(q)=\pm(E(k+q\mathrm{Cos}\alpha)-E(k)); \quad \mathrm{Cos}\alpha=\pm 1, \quad q>0$$

The both dependences are represented in Figure 7, where $x = ql$ was introduced.

The case of $k \neq 0$ with filling factor $v = v^2 < 1$ represents interest because in this region of parameters a metastable dielectric liquid phase does exist. It is formed by the Bose-Einstein condensed magnteoexcitons with $kl \sim 3-4$ and with different from zero motional dipole moments $\vec{\rho} = [\vec{k} \times \vec{z}]l^2$. This state was revealed in Ref. [9] considering the system of electrons and holes on their lowest Landau levels, without addressing to excited Landau levels (ELLs), but taking into account the coherent excited states, when one e–h pair exits from the condensate leaving all another pairs in their coherent pairing state.

The correlation energy was calculated beyond the Hartree–Fock–Bogoliubov approximation (HFBA)[9, 33] in the frame of Keldysh–Kozlov–Kopaev method[22, 23] using the Nozieres Comte approach.[17, 18]

The Bose-Einstein condensed magnetoexcitons moving as a whole with wave vector $\vec{k}$ and with parallel each other motional dipole moments $\vec{\rho}$ have a significant polarizability which gives rise to attractive interaction between them and which lowers on the energy scale the values of the chemical potential and of the mean energy per one e–h pair. But this lowering is not monotonous and at some value of the filling factor v_m^2 the relative minima on the corresponding curves appear with positive compressibilities in their vicinity. The relative minimum on the chemical potential curve depends essentially on the damping of magnetoexciton level. It was investigated in the Ref. [10] and is represented in Figure 8.

If the average filling factor v^2 is less than v_m^2 the dielectric liquid phase will exist in the form of droplets with optimal concentration inside them $n_{ex} = v_m^2/2\pi l_0^2$ corresponding to filling factor v_m^2.

The collective elementary excitations are calculated in the conditions $kl \sim 3-4$ and $v^2 \approx v_m^2$, when the ground state of the magnetoexcitons is similar with the metastable dielectric liquid phase.

Even in collinear geometry the diagonal self-energy parts $\Sigma_{ii}(\vec{K} + \vec{q}, \omega)$ with $i = 1, 2, 3, 4$ and $kl = 3, 6$ can not be calculated analytically at arbitrary values of the relative wave vector $\vec{q}$. By this reason we will obtain the analytical expressions in the case $kl \approx 3.6$ and $ql \leq 1 < kl$ using a series expansions on the small values $ql < 1$ as compared with $kl \approx 3.6$.

Up till now we have discussed the energy spectrum of a Bose-Einstein condensed magnetoexcitons in pure ideal conditions which take place in the case $k = 0$, when the interactions in the electron–hole system are reciprocally compensated at arbitrary values of the filling factor $v^2 \neq 0$, as well as in the case $k \neq 0$, when the nonlinearity is completely neglected putting $v = 0$. In the last case taking into account the nonlinearity $v^2 \neq 0$ we can observe its unusual influence on the earlier discussed energy spectrum leading to its qualitative new and principle changes. They are

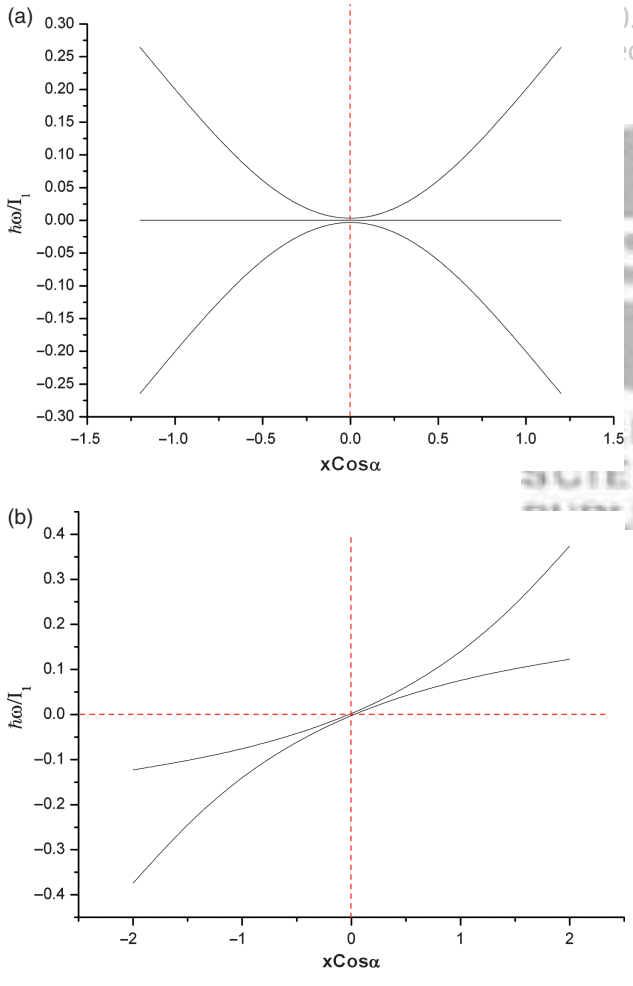

Fig. 7. The energy spectrum of elementary excitations of magnetoexcitons and acoustical plasmons in the case when concentrations corrections haven't been taken into account. (a) The wave vector of BEC magnetoexcitons equal to 0. (b) The wave vector k is different from zero, but the filling factor equals to zero.

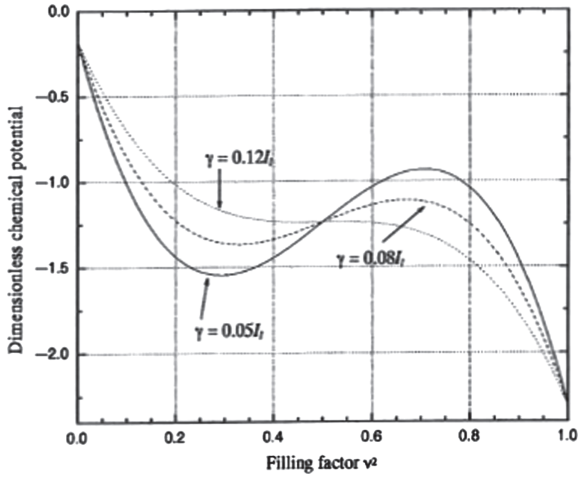

Fig. 8. Chemical potencial of the BEC-ed magnetoexcitons taking into account the damping γ of their energy levels.

different from the simple additions of the concentration corrections to the exciton branches of spectrum as one could expect on the base of a simple perturbation theory. Instead of it the influence of the concentration terms proportional to u^2v^2 entering into the compositions of the self-energy parts σ_{11}, σ_{22} and σ_{44} happens to be much more important. The self-energy parts contain the different linear on $\tilde{\omega}$ expressions of the type $L_i(\tilde{\omega})=\tilde{\omega}+\tilde{\mu}-\tilde{E}(y+x\mathrm{Cos}\alpha)$ which appear in the forms $A_i/L_i(\tilde{\omega})$ and determine the concentration corrections. For simplicity we will demonstrate their influence taking into account only the denominators in the first power. The self-energy parts σ_{11} and σ_{22} contain also such denominators in power two of the forms $B_i/(L_i(\tilde{\omega}))^2$, but these terms for simplicity were neglected in the numerical calculations. The presence of the unknown frequency $\tilde{\omega}$ in the denominators side by side with another term in numerators leads to the increasing of the order of the dispersion equation and of the number of the energy spectrum branches. In our concrete case the order of dispersion equation is doubled and instead of three branches of the energy spectrum we are dealing with six branches. Two of them are acoustical plasmon branches with energies proportional to the perturbation theory parameter $v^2(1-v^2)$ and with different $\pm$ signs. It was natural to expect the appearance of these two branches of acoustical plasmon spectrum and the same takes place with the optical plasmon spectrum. Unusual behavior happens with the exciton energy and quasienergy branches which become doubled undergoing each of them a bifurcation. The new branches have the form of the previous exciton branch plus or minus one additional of amount approximately equal to the energy of the acoustical plasmon with wave vector different from the wave vector of the exciton elementary excitation by the condensate wave vector k. The same change takes place with the quasienergy exciton branch. The neglected denominator in power two could create exciton branch with two acoustical plasmons. The Bose-Einstein condensation with $k\neq 0$ means that the e–h system is moving as regards the laboratory reference frame with a velocity equal to the group velocity V_g of the magnetoexcitons, reflected in the Figure 9.

It means that the terms $\hbar\vec{V}_g\vec{q}$ will appear in the dispersion relations for all three branches. To create the exciton-type collective elementary excitations when the ground state of the system is a dielectric liquid phase with negative values of the chemical potential μ it is necessary to eliberate an exciton from the liquid communicating it an amount of energy at least equal to $|\mu|$. This values $|\mu|$ are equal to $0.31I_l$ and $0.69I_l$ at the filling factors v^2 equal to 0.028 and 0.28 correspondingly. Because the concentration corrections to the energy spectrum in our case appear in the form of acoustical plasmon energy $\hbar\omega_{AP}$ proportional to the infinitesimal parameter $v^2(1-v^2)$ two exciton branches have approximately the energies $|\mu|\pm\hbar\omega_{AP}$.

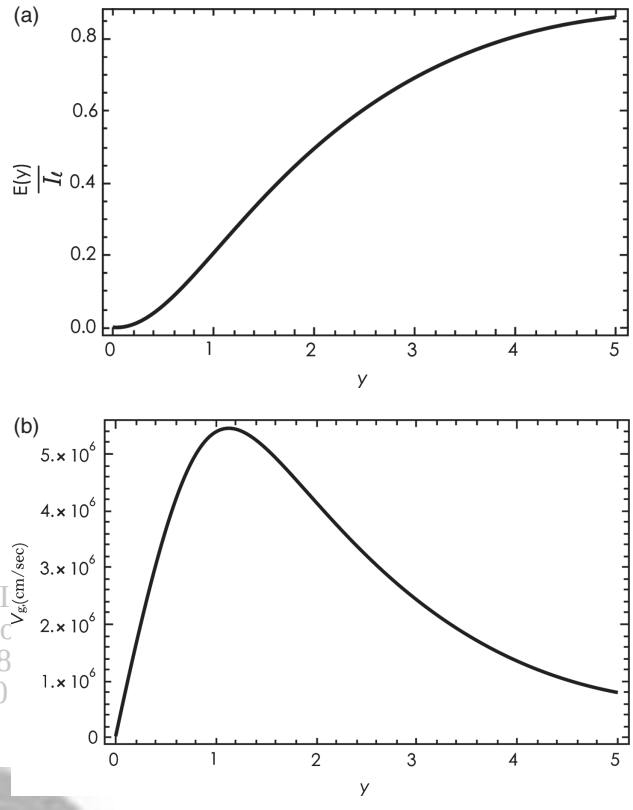

Fig. 9. (a) The dispersion law of the magnetoexciton. (b) The group velocity $V_g(k)$ of the magnetoexciton; $y=kl$.

The exciton and plasmon quasienergy branches can be obtained from the exciton and plasmon energy branches by two successive reflections as regards two coordinate axes. These properties can be observed on the Figure 10.

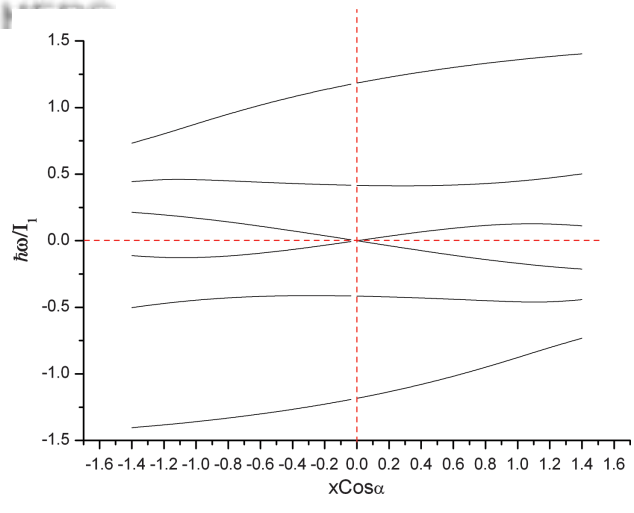

Fig. 10. The energy spectrum of elementary excitations of magnetoexcitons and acoustical plasmons in the case when filling factor of the lowest Landau levels equals to $v^2=0.28$. The dimensionless wave vector of the Bose-Einstein condensed magnetoexcitons equals to 3.6.

5. INFLUENCE OF THE EXCITED LANDAU LEVELS ON THE TWO-DIMENSIONAL ELECTRON–HOLE SYSTEM IN A STRONG PERPENDICULAR MAGNETIC FIELD

The purpose of the present part is the detailed study of the influence of virtual quantum transitions of the Coulomb interacting particles from the LLLs to all excited Landau levels (ELLs). We consider three aspects of the problem: the influence on the chemical potential of the BEC-ed magnetoexcitons, on the energy per pair in the composition of EHL and electron–hole droplets (EHD), as well as the influence on the wave function and on the energy level of a single magnetoexciton. The first steps in this direction were made in Ref. [10].

The full Hamiltonian consists from zero order Hamiltonian H_0 describing the Landau quantization of free quasiparticles and from the Hamiltonian H_{Coul} reflected their Coulomb interaction as follows

$$H = H_0 + H_{\text{Coul}} \tag{95}$$

where

$$\begin{aligned} H_{\text{Coul}} &= \frac{1}{2}\sum_{p,q,s}\sum_{n,m,n',m'}[F_{e-e}(p,n;q,m;p-s,n';q+s,m') \\ &\times a^{\dagger}_{n,p}a^{\dagger}_{m,q}a_{m',q+s}a_{n',p-s} + F_{h-h}(p,n;q,m;p-s,n'; \\ &\quad q+s,m')b^{\dagger}_{n,p}b^{\dagger}_{m,q}b_{m',q+s}b_{n',p-s}] \\ &- \sum_{p,q,s}\sum_{n,m,n',m'} F_{e-h}(p,n;q,m;p-s,n';q+s,m') \\ &\quad \times a^{\dagger}_{n,p}b^{\dagger}_{m,q}b_{m',q+s}a_{n',p-s} \end{aligned} \tag{96}$$

The operators of second quantization $a^{\dagger}_{np}, a_{np}$ for electron and $b^{\dagger}_{mq}, b_{mq}$ for holes describe their creation and annihilation in two-dimensional quantum states in a strong perpendicular magnetic field.[38] In Landau gauge they are characterized by the numbers n, m of Landau levels and by wave numbers p, q enumerating the N-fold degenerate single particle states. $N = S/(2\pi l^2)$, where S is surface area.

The matrix elements of the two-particle Coulomb interaction V_{12} are determined as:

$$\begin{aligned} &F_{i-j}(p,n;q,m;p-s,n';q+s,m') \\ &= \int d\vec{\rho}_1 \int d\vec{\rho}_2 \psi^{i*}_{n,p}(\vec{\rho}_1)\psi^{j*}_{m,q}(\vec{\rho}_2)V_{12}\psi^{i}_{n',p-s}(\vec{\rho}_1)\psi^{j}_{m',q+s}(\vec{\rho}_2) \end{aligned} \tag{97}$$

From the Coulomb interaction part we separate the Coulomb interaction within the LLLs denoted as H^{LLL}_{Coul} and the terms of the types

$$\begin{aligned} &F_{i-j}(0,p;0,q;n,p-s;m,q+s) \\ &F_{j-i}(n,p;m,q;0,p-s;0,q+s) \end{aligned} \tag{98}$$

which describe the virtual quantum transitions from the initial states $n=m=0$ to excited Landau levels (ELLs) $n,m\neq 0$ as well as their return back. All these matrix elements were calculated starting with the quantum numbers $n=0,1,2,3,4$. On this base the generalized formulas, were deduced for arbitrary values of n and m. The cumbersame expressions are dropped. The part of Coulomb interaction containing the matrix elements (98) is named as $H^{\text{ELL}}_{\text{Coul}}$. The another parts of (96) are neglected.

The terms of the type (98) were excluded from the Hamiltonian (96) by the aid of unitary transformation[39] $\hat{U} = e^{i\hat{S}}$, where $\hat{S} = \hat{S}$ and is determined from the equation

$$i[\hat{H}_0, \hat{S}] + H^{ELL}_{\text{Coul}} = 0 \tag{99}$$

The new transformed Hamiltonian H_{eff} is determined as

$$\begin{aligned} H_{\text{eff}} &= {}_{ELL}\langle 0|e^{-iS}\hat{H}e^{is}|0\rangle_{ELL} \\ &\cong H^{LLL}_{\text{Coul}} + \frac{i}{2}{}_{ELL}\langle 0|[H^{ELL}_{\text{Coul}}, \hat{S}]|0\rangle_{ELL} \end{aligned} \tag{100}$$

Here the average is made using the vacuum state for ELL named as $|0\rangle_{ELL}\cdot H_{\text{eff}}$ has the final form

$$\begin{aligned} \hat{H}_{\text{eff}} &= -\mu_e\sum_p a^{\dagger}_p a_p - \mu_h\sum_p b^{\dagger}_p b_p + H^{LLL}_{\text{Coul}} \\ &- \frac{1}{2}\sum_{p,q,s}\phi_{e-e}(p,q,s)a^{\dagger}_p a^{\dagger}_q a_{q+s}a_{p-s} \\ &- \frac{1}{2}\sum_{p,q,s}\phi_{h-h}(p,q,s)b^{\dagger}_p b^{\dagger}_q b_{q+s}b_{p-s} \\ &- \sum_{p,q,s}\phi_{e-h}(p,q,s)a^{\dagger}_p b^{\dagger}_q b_{q+s}a_{p-s} \end{aligned} \tag{101}$$

Here the chemical potentials μ_e and μ_h for electrons and holes are included.

The matrix elements $\phi_{i-j}(p,q,s)$ are determined by the formulas[37]

$$\begin{aligned} \phi_{i-j}(p,q,z) &= \sum_{n,m}\frac{\phi_{i-j}(p,q,z;n,m)}{n\hbar\omega_{ci} + m\hbar\omega_{cj}} \\ \phi_{i-j}(p,q,z;n,m) &= \sum_t F_{i-j}(p,0;q,0;p-t,n;q+t,m) \\ &\times F_{i-j}(p-t,n;q+t,m;p-z,0;q+z,0) \end{aligned} \tag{102}$$

Now we will discuss the influence of the supplementary indirect interaction on the ground state energies of two collective phases formed by 2D e–h system in a strong perpendicular magnetic field. One of them is the metallic type electron–hole liquid (EHL) and another one is the Bose-Einstein condensation (BEC) of magnetoexcitons on the single-particle state with wave vector $\vec{k}$. Both of them will be discussed below in Hartree–Fock approximation (HFA). Considering the EHL we started with the effective Hamiltonian (101) but without chemical potential μ_e

and μ_h. We calculated the ground state energy at $T=0$, when the average values of the electron and hole numbers on the LLLs are equal to their filling factor v^2

$$\langle a_p^\dagger a_p\rangle=\langle b_p^\dagger b_p\rangle=v^2 \tag{103}$$

Applying the Wick theorem we obtained the ground state energy in HFA

$$E_{\mathrm{EHL}}=-N_{e-h}\left[v^2 I_l+v^2(2A-B)\right],\quad N_{e-h}=Nv^2 \tag{104}$$

The coefficient A and B are determined in the case of electrons and holes with equal masses $m_e=m_h$ and cyclotron frequencies $\omega_{\mathrm{ce}}=\omega_{\mathrm{ch}}=\omega_c$. In this case one can write[37,40]

$$\begin{aligned}A &= A_{i-j}=\sum_p \phi_{i-j}(p,q,0)\\ &= \frac{I_l^2}{\pi\hbar\omega_c}\sum_{n\geq 1}^{\infty}\sum_{m\geq 1}^{\infty}\frac{(n+m-1)!}{2^{n+m}n!m!(n+m)}\\ &= \frac{I_l^2}{\pi\hbar\omega_c}S;\quad S=0.481\end{aligned} \tag{105}$$

Here I_l is the ionization potential of magnetoexciton within the LLLs approximation and equals to $(e^2)/(\varepsilon l)\sqrt{\pi/2}$, where l is magnetic length and ε is the background dielectric constant. In a similar way we have[40,37]

$$\begin{aligned}B &= B_{i-i}=\sum_s \phi_{i-i}(p,p-s,s)\\ &= \frac{2I_l^2}{\pi\hbar\omega_c}\sum_{n\geq 1}^{\infty}\sum_{m\geq 1}^{\infty}\frac{A_{n+m}^{n+m,n+m}}{2^{n+m}n!m!(n+m)}=\frac{2I_l^2}{\pi\hbar\omega_c}T\\ T &= 0.2161\end{aligned} \tag{106}$$

The energy per one e–h pair E_{EHL} in the componence of EHL in units of I_l equals to

$$\begin{aligned}\frac{E_{\mathrm{EHL}}}{I_l} &= -v^2\left[1+\frac{2I_l}{\pi\hbar\omega_c}(S-T)\right]\\ &= -v^2\left(1+0.168\frac{I_l}{\hbar\omega_c}\right)\end{aligned} \tag{107}$$

The lowest energy is achieved at filling factor $v^2=1$ and it determines the energy per pair inside the electron–hole droplet (EHD) equal to

$$E_{\mathrm{EHD}}/I_l=-(1+0.168r);\quad r=I_l/\hbar\omega_c \tag{108}$$

The ratio r must be less than 1, so to obey the condition of a strong magnetic field. In the Figure 11 this value was put 1/2. Now we will discuss the BEC of magnetoexcitons on the single particle state with wave vector $k\neq 0$, in Hartree–Fock–Bogoliubov approximation (HFBA). As was demonstrated in the papers[8,9] the BEC can be introduced into the starting Hamiltonian (101) by the help of the canonical transformation

$$a_p=u\alpha_p+v\left(p-\frac{k_x}{2}\right)\beta_{k_x-p}^\dagger,\quad b_p=u\beta_p-v\left(\frac{k_x}{2}-p\right)\alpha_{k_x-p}^\dagger \tag{109}$$

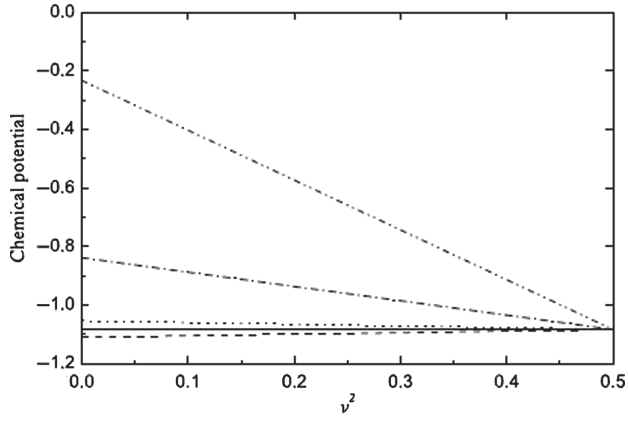

Fig. 11. Chemical potential in units of binding energy I_l versus dimensionless wave vector kl for $I_l/\hbar\omega_c=1/2$. Solid line: energy per e–h pair in EHD phase; dashed line: chemical potential of condensed excitons with $k=0$; dotted line: the same, but for $kl=0.5$; dash-dotted line: the same, but for $kl=1$; dash-dot-dot line: the same, but for $kl=3.6$. Reprinted with permission from [37], S. A. Moskalenko et al., *Physica E* 39/1, 137 (**2007**). © 2007, Elsevier.

where the coefficients are

$$v(t)=ve^{-ik_y tl^2},\quad u^2+v^2=1 \tag{110}$$

Being transcribed in the operators $\alpha_p^\dagger,\beta_p^\dagger,\alpha_p,\beta_p$ the Hamiltonian (101) after its normal ordering will generate in the Hartree–Fock–Bogoliubov approximation the quadratic expression H_2 similar with the quadratic Hamiltonian (39) of the paper,[9] which contains the coefficients $E(\mathbf{k},v^2,\mu)$ and $\psi(\mathbf{k},v^2,\mu)$ defined by the formulas (40) and (41).[9] However, contrary to the quadratic expression of the paper[9] the generalized quadratic Hamiltonian H_2 contains supplementary terms. Dropping the intermediary cumbersome calculations we can write the new Hamiltonian H_2 in a special case $m_e=m_h$, $\omega_{ce}=\omega_{ch}=\omega_{c0}$, $\mu_e=\mu_h=\mu/2$.

$$\begin{aligned}H_2 &= \sum_p\left[E(\mathbf{k},v^2,\mu)+(B-2A)v^2(1-2v^2)\right.\\ &\quad\left.+2v^2(1-v^2)\Delta(k)\right](\alpha_p^+\alpha_p+\beta_p^+\beta_p)\\ &+\sum_p\left[uv\left(\frac{k_x}{2}-p\right)\beta_{k_x-p}\alpha_p\right.\\ &\quad\left.+uv\left(p-\frac{k_x}{2}\right)\alpha_p^+\beta_{k_x-p}^+\right]\\ &\times\left\{-\psi(k,v^2,\mu)+2v^2(B-2A+\Delta(k))-\Delta(k)\right\}\end{aligned} \tag{111}$$

Here $\Delta(k)$ is determined by the sum[37]

$$\begin{aligned}\Delta(k) &= \sum_z \phi_{e-h}(p,k_x-p,z)e^{-ik_y zl^2}\\ &= \frac{I_l^2 e^{-k^2l^2}}{\pi}\sum_{n,m}\frac{(kl)^{2|n-m|}}{n!m!2^{|n-m|}(n\hbar\omega_{ce}+m\hbar\omega_{ch})}\end{aligned}$$

$$\times\left[\frac{\Gamma((m+n+|n-m|+1)/2)}{\Gamma(|n-m|+1)}\right]^2 \times\left[{}_1F_1\left(\frac{|n-m|+1-(n+m)}{2};|n-m|+1;\frac{(kl)^2}{2}\right)\right]^2 \quad (112)$$

In the point $k=0$ and $\omega_{ce}=\omega_{ch}=\omega_c$ we have[37]

$$\Delta(0)=\frac{2I_l^2}{\pi\hbar\omega_c}(\pi\ln 2-2G)=\frac{2I_l^2}{\pi\hbar\omega_c}0.344 \quad (113)$$

where G is the Catalana constant (≈ 0.915966).[40]

Following the formulas (40) and (41)[9] we have

$$E(\mathbf{k},v^2,\mu)=2v^2u^2I_{\rm ex}(k)+I_l(v^4-v^2u^2)-\frac{\mu}{2}(u^2-v^2)$$
$$\psi(\mathbf{k},v^2,\mu)=2v^2I_l+I_{\rm ex}(k)(1-2v^2)+\mu \quad (114)$$

The last brackets in (111) was put equal to zero, what leads to the compensation of dangerous diagrams of H_2. They describe the spontaneous creation of the e–h pairs from new vacuum state and their annihilation in this vacuum. In such a way the chemical potential μ in the HFBA was determined as follows:

$$\mu^{\rm HFB}=-\tilde{I}_{\rm ex}(k)+2v^2(B-2A+\tilde{I}_{\rm ex}(k)-I_l)$$
$$=-\tilde{I}_{\rm ex}(k)+2v^2(B-2A+\Delta(k)-E(k)) \quad (115)$$

Here the renormalized ionization potential of magnetoexcitons $\tilde{I}_{\rm ex}(k)$ was introduced:

$$\tilde{I}_{\rm ex}(k)=I_{\rm ex}(k)+\Delta(k)$$
$$I_{\rm ex}(k)=I_l-E(k)$$
$$E_{\rm ex}(k)=-I_{\rm ex}(k) \quad (116)$$

Our results are represented on Figure 1. The solid line determines the energy per e–h pair in the composition of EHD. The dashed lines represent the dependences of chemical potentials $\mu(k,v^2)$ for different values of k versus the filling factor v^2. Only the values $v^2\le 1/2$ are considered following the criteria of the elaborated theory.[9,37] At small values of $k(kl<0.5)$ the plotted dependences $\mu(k,v^2)$ are increasing and the corresponding ground states are stable in HFBA. At the same time we can notice that these values $\mu(k,v^2)$ lie on the energy scale not so far from the energy per e–h pair in EHD. It means that these states can coexist being realized in different sites of the sample. From another hand as was shown in Ref. [9] at considerable values of wave vectors and motional dipole moments the Bose-Einstein condensed magnetoexcitons can form a metastable dielectric liquid phase. This possibility was revealed taking into account the correlation energy behind the HFBA and the coherent excited states.[25] Their influence diminish in the region of small wave vectors, where the influence of ELLs is maximal. In such a way there are two regions of wave vectors with the predominant influence of different excited states. In Ref. [41] was suggested the coexistence of degenerate Bose-gas with small wave vectors and of the drops of metastable dielectric liquid phase formed by Bose-Einstein condensed magnetoexcitons with considerable values of wave vectors $k(kl\sim 3-4)$. Taking into account the ELLs we arrived to the conclusion that the metallic-type electron–hole droplet can coexist with the previous two dielectric gaseous and liquid phases.

6. COLLECTIVE ELEMENTARY EXCITATIONS OF TWO-DIMENSIONAL MAGNETOEXCITONS IN THE BOSE-EINSTEIN CONDENSATION STATE WITH WAVE VECTOR $k=0$

The Hamiltonian of the Coulomb interaction of the electrons and holes in the frame of lowest Landau levels (LLLs) has the form:

$$\hat{H}=\frac{1}{2}\sum_{\vec{Q}}W_{\vec{Q}}[\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})-\hat{N}_e-\hat{N}_h]-\mu_e\hat{N}_e$$
$$-\mu_h\hat{N}_h+\hat{H}_{\rm suppl} \quad (117)$$

where $W_{\vec{Q}}$ is the Fourier transform of the Coulomb interaction in the frame of LLLs, $\hat{N}_e$ and $\hat{N}_h$ are the operators of the numbers of electrons and holes on the LLLs. They were determined above. $\hat{H}_{\rm suppl}$ is the supplementary indirect attractive interaction between the particle lying on the lowest Landau levels (LLLs) in view of their virtual transitions on the excited Landau levels (ELLs) and their return back:[37]

$$H_{\rm suppl}=-\frac{1}{2}\sum_{p,q,s}\phi_{e-e}(p,q;s)a_p^\dagger a_q^\dagger a_{q+s}a_{p-s}$$
$$-\frac{1}{2}\sum_{p,q,s}\phi_{h-h}(p,q;s)b_p^\dagger b_q^\dagger b_{q+s}b_{p-s}$$
$$-\sum_{p,q,s}\phi_{e-h}(p,q;s)a_p^\dagger b_q^\dagger b_{q+s}a_{p-s} \quad (118)$$

Here the creation and annihilation operators $a_p^\dagger, a_p$ for electrons and $b_q^\dagger, b_q$ for holes were introduced. The matrix elements of indirect interaction $\phi_{i-j}(p,q,z)$ are described by the common expressions[37]

$$\phi_{i-j}(p,q,s)=\sum_{n,m}\frac{\phi_{i-j}(p,q,z;n,m)}{n\hbar\omega_{ci}+m\hbar\omega_{cj}} \quad (119)$$

In the case of electron–electron and hole–hole interaction the expression (119) has the form:[37]

$$\phi_{i-i}(p,q,z;n,m)$$
$$\cong\sum_{t,\kappa,\sigma}W_{t,k}W_{z-t,\sigma}\exp\left(i\kappa(p-q-t)l^2\right)$$
$$\times\exp\left(i\sigma(p-q-t-z)l^2\right)(t+i\kappa)^{n+m}(t-z+i\sigma)^{n+m} \quad (120)$$

but in the case of electron–hole interaction is:

$$\phi_{e-h}(p,q,z;n,m) \cong \sum_{t,\kappa,\sigma} W_{t,k} W_{z-t,\sigma} \exp\big(i(\kappa+\sigma)(p+q)l^2\big) \times (t+i\kappa)^n (t-i\kappa)^m (t-z+i\sigma)^n (t-z-i\sigma)^m \quad (121)$$

where

$$W_{s,\kappa} = \frac{2\pi e^2}{\varepsilon_0 S\sqrt{s^2+k^2}} e^{-((s^2+k^2)l^2)/2} \qquad W_{s,k} = W_{-s,-k} = W_{-s,k} = W_{s,-k} \quad (122)$$

The Hamiltonian (119) is a hermitian conjugate form, if the requirements are fulfilled

$$\phi^*_{i-j}(p-s,q+s;-s) = \phi_{i-j}(p,q;s), \quad i,j = e,h \quad (123)$$

Their Fourier transforms are

$$\psi_{i-j}(s,\sigma) = \sum_{\kappa} \widetilde{\phi}_{i-j}(s,\kappa)\exp(i\kappa\sigma l^2) \quad (124)$$

The Hamiltonian (118) written in the terms of the single particle operators $a^\dagger_p, a_p, b^\dagger_p, b_p$ has been transformed to the form containing the two-particle operators of the electrons and holes densities $\hat{\rho}_e(\vec{Q})$ and $\hat{\rho}_h(\vec{Q})$ of the type

$$\hat{\rho}_e(\vec{Q}) = \sum_t e^{iQ_y t l^2} a^\dagger_{t-Q_x/2} a_{t+Q_x/2}$$
$$\hat{\rho}_h(\vec{Q}) = \sum_t e^{iQ_y t l^2} b^\dagger_{t+Q_x/2} b_{t-Q_x/2} \quad (125)$$

The relations between two sets of operators are:

$$a^\dagger_{p-s/2} a_{p+s/2} = \frac{1}{N}\sum_\kappa \hat{\rho}_e(s,\kappa)\exp(-i\kappa p l^2)$$
$$a^\dagger_p a_{p-s} = \frac{1}{N}\sum_\kappa \hat{\rho}_e(-s,\kappa)\exp\left(-i\kappa p l^2 + \frac{is\kappa}{2}l^2\right)$$
$$a^\dagger_q a_{q+s} = \frac{1}{N}\sum_\kappa \hat{\rho}_e(s,\kappa)\exp\left(-i\kappa q l^2 - \frac{is\kappa}{2}l^2\right) \quad (126)$$

where $N = S/(2\pi l^2)$, S is the layer surface area and l is the magnetic length. Here the δ-symbol Kronecker was used

$$\frac{1}{N}\sum_p \exp(ip(\sigma-\kappa)l^2) = \delta_{kr}(\sigma,\kappa) \quad (127)$$

The Hamiltonian of supplementary indirect attractive interaction (119) has the form:

$$H_{\text{suppl}} = \frac{1}{2}B_{i-i}\hat{N} - \frac{1}{2N}\sum_{s,\sigma}\psi_{i-i}(s,\sigma) \times \big[\hat{\rho}_e(-s,-\sigma)\hat{\rho}_e(s,\sigma) + \hat{\rho}_h(-s,-\sigma)\hat{\rho}_h(s,\sigma)\big] - \frac{1}{N}\sum_{s,\sigma}\psi_{e-h}(s,\sigma)\hat{\rho}_e(-s,-\sigma)\hat{\rho}_h(-s,-\sigma) \quad (128)$$

Instead of density operators for electrons and holes we can introduce their in-phase and in opposite-phase linear combinations

$$\hat{\rho}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) - \hat{\rho}_h(-\vec{Q})$$
$$\hat{D}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q})$$
$$\hat{\rho}_e(\vec{Q}) = \frac{1}{2}\Big[\hat{\rho}(\vec{Q}) + \hat{D}(\vec{Q})\Big]$$
$$\hat{\rho}_h(\vec{Q}) = \frac{1}{2}\Big[\hat{D}(-\vec{Q}) - \hat{\rho}(-\vec{Q})\Big] \quad (129)$$

They lead to the following relations

$$\hat{\rho}_e(-\vec{Q})\hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q})\hat{\rho}_h(\vec{Q}) = \frac{1}{2}\big[\hat{\rho}(-\vec{Q})\hat{\rho}(\vec{Q}) + \hat{D}(-\vec{Q})\hat{D}(\vec{Q})\big]$$
$$\sum_Q \psi_{e-h}(Q)\big[\hat{\rho}(-\vec{Q})\hat{D}(\vec{Q}) - \hat{D}(-\vec{Q})\hat{\rho}(\vec{Q})\big] = \sum_Q \psi_{e-h}(Q)\big[\hat{\rho}(-\vec{Q})\hat{D}(\vec{Q}) - \hat{D}(\vec{Q})\hat{\rho}(-\vec{Q})\big] = 0$$

and to the final expression

$$H_{\text{suppl}} = \frac{1}{2}B_{i-i}\hat{N} - \frac{1}{4N}\sum_Q V(Q)\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) - \frac{1}{4N}\sum_Q U(Q)\hat{D}(\vec{Q})\hat{D}(-\vec{Q}) \quad (130)$$

where

$$U(Q) = \psi_{i-i}(Q) + \psi_{e-h}(Q)$$
$$V(Q) = \psi_{i-i}(Q) - \psi_{e-h}(Q) \quad (131)$$

The estimations show that

$$U(0) = 2A_{i-i}; \quad V(0) = 0; \quad \frac{1}{N}\sum_{\vec{Q}} U(\vec{Q}) = B_{i-i} + \Delta(0)$$

It means that one can suppose the dependences

$$U(\vec{Q}) \cong U(0)e^{-(Q^2l^2)/2}; \quad V(\vec{Q}) \cong V(0) = 0 \quad (132)$$

The starting Hamiltonian in QATA has the form

$$\hat{\mathcal{H}} = \frac{1}{2}\sum_{\vec{Q}} W_{\vec{Q}}\big[\rho(\vec{Q})\rho(-\vec{Q}) - \hat{N}_e - \hat{N}_h\big] - \mu_e\hat{N}_e - \mu_h\hat{N}_h - \tilde{\eta}\sqrt{N}\big(e^{i\varphi}d^\dagger(k) + e^{-i\varphi}d(k)\big) + \frac{1}{2}B_{i-i}\hat{N} - \frac{1}{4N}\sum_Q V(Q)\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) - \frac{1}{4N}\sum_Q U(Q)\hat{D}(\vec{Q})\hat{D}(-\vec{Q}) \quad (133)$$

The density fluctuation operators (129) with different wave vectors P and Q do not commute, which is related with

the helicity or spirality accompanying the presence of the strong magnetic field. They are expressed by the phase factors in the structure of operators (125). The vector-product of two 2D wave vectors P and Q and its projection on the direction of the magnetic field appears, when their commutations are calculated. These properties considerably influence structure of the equations of motion for the operators and determine new aspects of the 2D electron–hole (e–h) physics.

The equation of motion for the creation and annihilation operators $d^+(\vec{P}), d(\vec{P})$ and for the density fluctuation operators (73) will be deduced, when the BEC takes place on the state $k=0$. They are:

$$
\begin{aligned}
&i\hbar\frac{d}{dt}d(\vec{P})\\
&=[d(\vec{P}),\hat{\mathscr{H}}]\\
&=(-\bar{\mu}+E(\vec{P})-\Delta(\vec{P}))d(\vec{P})\\
&\quad-2i\sum_{\vec{Q}}\tilde{W}(\vec{Q})\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q})\\
&\quad-\frac{1}{N}\sum_{\vec{Q}}U(\vec{Q})\mathrm{Cos}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)D(\vec{Q})d(\vec{P}-\vec{Q})\\
&\quad-\tilde{\eta}\sqrt{N}e^{i\varphi}\delta_{kr}(\vec{P},0)+\tilde{\eta}e^{i\varphi}\frac{D(\vec{P})}{\sqrt{N}} \qquad (134)\\
&i\hbar\frac{d}{dt}d^{\dagger}(-\vec{P})\\
&=[d^{\dagger}(-\vec{P}),\hat{\mathscr{H}}]\\
&=(\bar{\mu}-E(-\vec{P})+\Delta(-\vec{P}))d^{\dagger}(-\vec{P})\\
&\quad+2i\sum_{\vec{Q}}\tilde{W}(\vec{Q})\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d^{\dagger}(-\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q})\\
&\quad+\frac{1}{N}\sum_{\vec{Q}}U(\vec{Q})\mathrm{Cos}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d^{\dagger}(-\vec{P}-\vec{Q})D(-\vec{Q})\\
&\quad+\tilde{\eta}\sqrt{N}e^{-i\varphi}\delta_{kr}(\vec{P},0)-\tilde{\eta}e^{-i\varphi}\frac{D(\vec{P})}{\sqrt{N}}\\
&i\hbar\frac{d}{dt}\hat{\rho}(\vec{P})\\
&\quad=[\hat{\rho}(\vec{P}),\hat{\mathscr{H}}]\\
&\quad=-i\sum_{\vec{Q}}\tilde{W}(\vec{Q})\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\\
&\qquad\times[\hat{\rho}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})+\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})]\\
&\qquad+\frac{i}{2N}\sum_{\vec{Q}}U(\vec{Q})\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\\
&\qquad\times\left[D(\vec{P}-\vec{Q})D(\vec{Q})+D(\vec{Q})D(\vec{P}-\vec{Q})\right]\\
&i\hbar\frac{d}{dt}\hat{D}(\vec{P})\\
&\quad=[\hat{D}(\vec{P}),\hat{\mathscr{H}}]\\
&\quad=-i\sum_{\vec{Q}}\tilde{W}(\vec{Q})\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)[\hat{\rho}(\vec{Q})\hat{D}(\vec{P}-\vec{Q})\\
&\qquad+\hat{D}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})]\\
&\qquad+\frac{i}{2N}\sum_{\vec{Q}}U(\vec{Q})\mathrm{Sin}\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)[\hat{D}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})\\
&\qquad+\hat{\rho}(\vec{P}-\vec{Q})\hat{D}(\vec{Q})]+2\tilde{\eta}\sqrt{N}\left[e^{-i\varphi}d(\vec{P})-e^{i\varphi}d^{\dagger}(-\vec{P})\right]
\end{aligned}
$$

Here

$$
\begin{aligned}
\tilde{\eta}&=(\tilde{E}_{\mathrm{ex}}(k)-\mu)v=(E(k)-\Delta(k)-\bar{\mu})v\\
\tilde{E}_{\mathrm{ex}}(k)&=E_{\mathrm{ex}}(k)-\Delta(k)=-I_l-\Delta(k)+E(k)\\
E_{\mathrm{ex}}(k)&=-I_l+E(k)\\
E(K)&=2\sum_{Q}W_Q\mathrm{Sin}^2\left(\frac{[K\times Q]_z l^2}{2}\right)\\
\bar{\mu}&=\mu+I_l;\quad v=v^2;\quad N_{\mathrm{ex}}=v^2N\\
\tilde{W}(Q)&=W_Q-\frac{1}{2N}V(Q)\\
\Delta(k)&=\sum_{s}\phi_{e-h}(p,-p-k_x,s)e^{-k_y s l^2}\\
&=\frac{1}{N}\sum_{Q}\psi_{e-h}(Q)\exp(i[k\times Q]_z l^2) \qquad (135)
\end{aligned}
$$

Following the equations of motion (134) we will introduce four interconnected retarded Green's functions at $T=0$[35,36]

$$
\begin{aligned}
G_{11}(\vec{P},t)&=\langle\langle d(\vec{P},t);\hat{X}^{\dagger}(\vec{P},0)\rangle\rangle\\
G_{12}(\vec{P},t)&=\langle\langle d^{\dagger}(-\vec{P},t);\hat{X}^{\dagger}(\vec{P},0)\rangle\rangle\\
G_{13}(\vec{P},t)&=\left\langle\left\langle\frac{\hat{\rho}(\vec{P},t)}{\sqrt{N}};\hat{X}^{\dagger}(\vec{P},0)\right\rangle\right\rangle \qquad (136)\\
G_{14}(\vec{P},t)&=\left\langle\left\langle\frac{\hat{D}(\vec{P},t)}{\sqrt{N}};\hat{X}^{\dagger}(\vec{P},0)\right\rangle\right\rangle
\end{aligned}
$$

They are determined by the relations (83), where $\hat{H}$ is the Hamiltonian (133).

The equations of motion for the Green's functions in a special case, when the BEC of magnetoexcitons takes place on the state with $k=0$, are:

$$
\begin{aligned}
&(\hbar\omega+i\delta+\bar{\mu}-E(P)+\Delta(P))G_1(P,\omega)\\
&=C-2i\sum_{Q}\tilde{W}(Q)\mathrm{Sin}\left(\frac{[P\times Q]_z l^2}{2}\right)\\
&\quad\times\langle\langle\rho(Q)d(P-Q)\,|X\rangle\rangle_{\omega}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{N}\sum_{Q}U(Q)\mathrm{Cos}\left(\frac{[P\times Q]_z l^2}{2}\right)\\
&\times\langle\langle D(Q)d(P-Q)\,|X\rangle\rangle_\omega+\tilde{\eta}G_4(P,\omega)e^{i\varphi}\\
&(\hbar\omega+i\delta-\bar{\mu}+E(-P)-\Delta(-P))G_2(P,\omega)\\
&=C+2i\sum_{Q}\tilde{W}(Q)\mathrm{Sin}\left(\frac{[P\times Q]_z l^2}{2}\right)\\
&\times\left\langle\left\langle d^{\dagger}(-P-Q)\rho(-Q)\,|X\right\rangle\right\rangle_\omega\\
&+\frac{1}{N}\left\langle\left\langle d^{\dagger}(-P-Q)D(-Q)\,|X\right\rangle\right\rangle_\omega-\tilde{\eta}G_4(P,\omega)e^{-i\varphi}\\
&(\hbar\omega+i\delta)G_3(P,\omega)\\
&=C-i\sum_{Q}\tilde{W}(Q)\mathrm{Sin}\left(\frac{[P\times Q]_z l^2}{2}\right)\\
&\times\left\langle\left\langle\frac{\rho(P-Q)\rho(Q)}{\sqrt{N}}+\frac{\rho(Q)\rho(P-Q)}{\sqrt{N}}\,|X\right\rangle\right\rangle_\omega\\
&+\frac{i}{2N}\sum_{Q}U(Q)\mathrm{Sin}\left(\frac{[P\times Q]_z l^2}{2}\right)\\
&\times\left\langle\left\langle\frac{D(P-Q)D(Q)}{\sqrt{N}}+\frac{D(Q)D(P-Q)}{\sqrt{N}}\,|X\right\rangle\right\rangle_\omega\\
&(\hbar\omega+i\delta)G_4(P,\omega)\\
&=C-i\sum_{Q}\tilde{W}(Q)\mathrm{Sin}\left(\frac{[P\times Q]_z l^2}{2}\right)\\
&\times\left\langle\left\langle\frac{D(Q)\rho(P-Q)}{\sqrt{N}}+\frac{D(P-Q)\rho(Q)}{\sqrt{N}}\,|X\right\rangle\right\rangle_\omega\\
&+\frac{i}{2N}\sum_{Q}U(Q)\mathrm{Sin}\left(\frac{[P\times Q]_z l^2}{2}\right)\\
&\times\left\langle\left\langle\frac{D(Q)\rho(P-Q)}{\sqrt{N}}+\frac{\rho(P-Q)D(Q)}{\sqrt{N}}\,|X\right\rangle\right\rangle_\omega\\
&+2\tilde{\eta}\left[e^{-i\varphi}G_1(P,\omega)-e^{i\varphi}G_2(P,\omega)\right]
\end{aligned}
\tag{137}
$$

Using Zubarev's procedure[36] for the Green's function we obtain a closed system of Dyson equations for the Green's functions in the forms:

$$
\sum_{j=1}^{4}G_{1j}(\vec{P},\omega)\Sigma_{jk}(\vec{P},\omega)=C_{1k};\quad k=1,2,3,4 \tag{138}
$$

The self-energy parts $\Sigma_{jk}(\vec{P},\omega)$ entering into the formulas (138) contain the different average values of the two-operator products. They were calculated using the ground state wave function $|\psi_g(0)\rangle$ taken with $k=0$ and have the expressions:

$$
\begin{aligned}
\langle D(\vec{Q})D(-\vec{Q})\rangle&=4u^2v^2N\\
\bar{\mu}&=-\Delta(0)+2v^2(B_{i-i}-2A_{i-i}+\Delta(0))\\
\langle D(\vec{Q})d(-\vec{Q})\sqrt{N}\rangle&=\langle d^{\dagger}(\vec{Q})D(-\vec{Q})\sqrt{N}\rangle=-2uv^3N\\
\langle d(0)\rangle&=\langle d^{\dagger}(0)\rangle=uv\sqrt{N};\quad \tilde{\eta}=-(\Delta(0)+\bar{\mu})v
\end{aligned}
\tag{139}
$$

All these averages are extensive values proportional to N or $\sqrt{N}$, they essentially depend on the small parameters of the types u^2v^2 or uv^3, or uv.

The cumbersome dispersion equation is expressed in general form by the determinant equation:

$$
\det|\Sigma_{ij}(\vec{P},\omega)|=0 \tag{140}
$$

Substituting the self-energy parts $\Sigma_{jk}(\vec{P},\omega)$ of the equations (138) in the formula (140), we will observe that the determinant equation (140) disintegrates in two independent equations. One of them concerns only to optical plasmons and has a simple form

$$
\Sigma_{33}(\vec{P};\omega)=0 \tag{141}
$$

whereas the second equation contains the self-energy parts Σ_{11}, Σ_{22}, Σ_{44}, Σ_{14}, Σ_{41}, Σ_{24}, Σ_{42} and the quasi-average constant $\tilde{\eta}$

$$
\begin{aligned}
&\Sigma_{11}(\vec{P};\omega)\Sigma_{22}(\vec{P};\omega)\Sigma_{44}(\vec{P};\omega)\\
&-\Sigma_{41}(\vec{P};\omega)\Sigma_{22}(\vec{P};\omega)\Sigma_{14}(\vec{P};\omega)\\
&-\Sigma_{42}(\vec{P};\omega)\Sigma_{11}(\vec{P};\omega)\Sigma_{24}(\vec{P};\omega)=0
\end{aligned}
\tag{142}
$$

The solutions of the dispersion Eq. (142) will be discussed in two limiting cases. One of them is the point $v^2=0$, where the system behaves as an ideal Bose gas and another case of $v^2\neq 0$.

All contributions to the self-energy parts contain the averages $\langle D(\vec{Q})D(-\vec{Q})\rangle, \langle D(\vec{Q})d(-\vec{Q})\sqrt{N}\rangle, \langle d(0)\rangle$ which do not vanish in the point $\vec{k}=0$. The 2D magnetoexciton system now is not at all a pure ideal gas. It was an ideal gas when the influence of ELLs was neglected. This unusual result was revealed for the first time by Lerner and Lozovik[5–7] and was confirmed by Paquet et al.[8] In the case $v^2=0$ because the vanishing of the averages (139) the self-energy parts become

$$
\begin{aligned}
\sigma_{11}(\vec{P},\omega)&=\hbar\omega-E(P);\quad \bar{\mu}+\Delta(0)=0\\
\sigma_{22}(\vec{P},\omega)&=\hbar\omega+E(-P);\quad \tilde{\eta}=0\\
\sigma_{33}(\vec{P},\omega)&=\hbar\omega;\quad \Delta(\vec{P})\approx\Delta(0)\\
\sigma_{44}(\vec{P},\omega)&=\hbar\omega;\quad v^2=0;\ \vec{k}=0
\end{aligned}
\tag{143}
$$

and the excitonic part of the dispersion relation as well the acoustic plasmon frequency look as

$$
\begin{aligned}
\hbar\omega_{\mathrm{ex}}(P)&=\pm E(P)\\
\hbar\omega_A(P)&=\hbar\omega_O(P)=0
\end{aligned}
\tag{144}
$$

The acoustical and optical plasmon branches have the frequencies equal to zero. This case is presented in Figure 12.

The solution of Eq. (142) is presented in Figure 13. The surprising results is that the spectrum of elementary excitations is doubled. We can say that this results reflects the

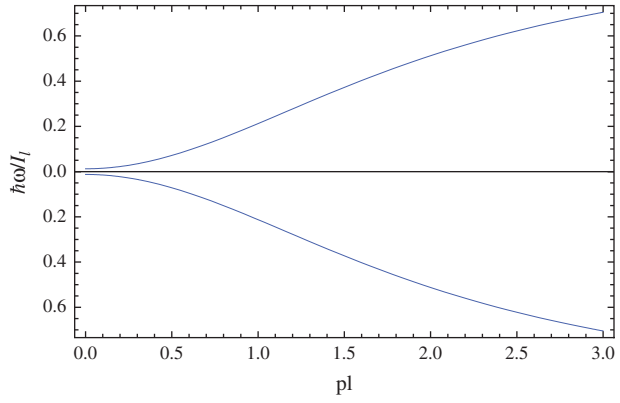

Fig. 12. The energy spectrum of elementary excitations of magnetoexcitons in the case when concentrations corrections haven't been taken into account, the filling factor equals to zero.

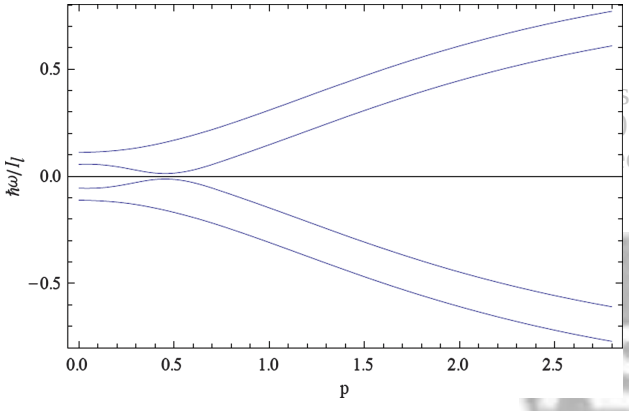

Fig. 13. The exciton branches of the energy spectrum of collective elementary excitations of the Bose-Einstein condensed magnetoexcitons on the wave vector $\vec{k}=0$, calculated in HFBA, using the self-energy parts of the Eq. (138) at the filling factor $v^2=0.1$.

existence of the quasi-energy complexes in system. One of curve is an elementary excitations of magnetoexcitons, but another one corresponds the elementary excitations of magnetoexcitons plus energy of Bose-Einstein condensed

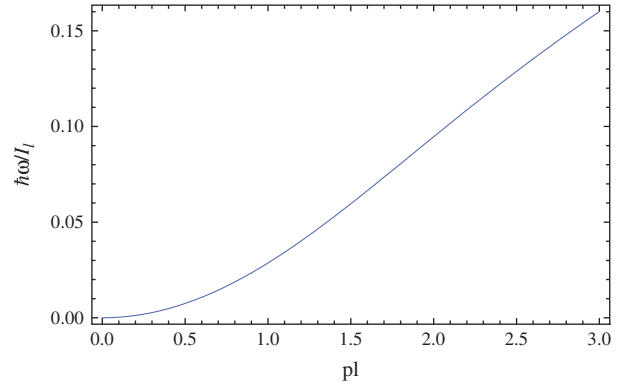

Fig. 14. The dispersion law of optical plasmon branch in the presence of the BEC of magnetoexcitons on the wave vector $\vec{k}=0$, calculated in HFBA, using the self-energy part $\Sigma_{33}(\vec{P},\omega)$ (138) and the filling factor $v^2=0.1$.

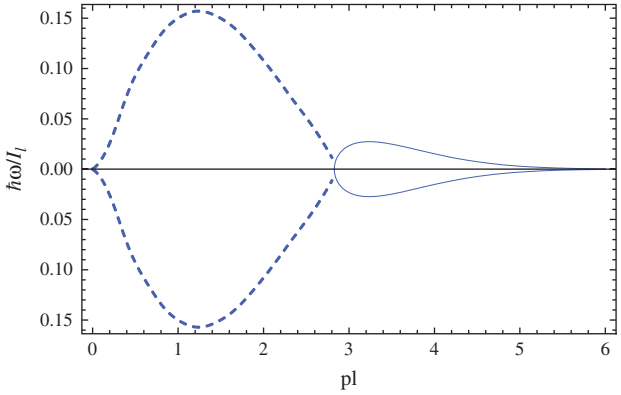

Fig. 15. The dispersion law of acoustical plasmon branch in the presence of the BEC of magnetoexcitons on the wave vector $\vec{k}=0$, calculated in HFBA, using the self-energy parts (138) and filling factor $v^2=0.1$.

magnetoexciton with wave vector $k=0$ determined by the value of the chemical potential.

The optical plasmon dispersion law, presented in Figure 14, is gapless with quadratic dependence in the range of small wave vectors and with saturation-type dependence in the remaining part of the spectrum. The acoustical plasmon branch, presented in Figure 15, reveals the absolute instability of the spectrum in the range of small and intermediary values of the wave vectors. In the remaining range of the wave vectors the acoustical plasmon branch has a very small real value of the energy spectrum tending to zero in the limiting case of great wave vectors.

7. CONCLUSIONS

A review paper presents the results received last years in the theory of Bose-Einstein condensation of two-dimensional magnetoexcitons. The system of high-density electron–hole pairs created on the surface of the semiconductor mono-layer in a strong perpendicular magnetic field is considered. A new metastable dielectric liquid phase formed by Bose-Einstein condensed 2D magnetoexcitons on the single-particle states with different from zero, sufficiently large values of the wave vector k was revealed in conditions, when the electrons and holes are situated on the lowest Landau levels (LLLs) and the interaction between the motional dipole moments in-plane parallel oriented perpendicular to the wave vector k gives rise to the attraction between magnetoexcitons. This phase is due to the polarizability of the BEC-ed magnetoexcitons, which appears taking into account the Anderson-type coherent excited states existing in the frame of LLLs. Its value is proportional to the coherence factor, being different from zero in the case $k\neq 0$, in spite of its vanishing value in the point $k=0$. The polarizability has a resonance denominator containing the magnetoexciton ionization potential $I_{ex}(k)$, which vanishes in the limit $k\to\infty$.

The chemical potential of the Bose-gas in these conditions was determined beyond the Hartree–Fock–Bogoliubov approximation (HFBA) taking into account the correlation energy. It has a nonmonotonous dependence on the filling factor v^2 of the LLLs and reveals a relative minimum with positive compressibility in its vicinity. This state does exist in the range of parameters $v^2 \approx \mathrm{Sin}^2 v$ and $kl \sim 3-4$, where l is magnetic length. All these peculiarities disappear when the BEC takes place in the point $k=0$, when the 2D magnetoexcitons behave as an ideal Bose-gas, if the influence of the excited Landau levels (ELLs) is neglected.

The influence of the ELLs on the quantum states of the electrons and holes lying on the LLLs is due to their virtual quantum transitions to ELLs and return back. These virtual quantum transitions were taken into account in the frame of the second order perturbation theory considering the matrix elements of the Coulomb interaction calculated with the wave functions of the ELLs as a small perturbation in comparison with the matrix elements calculated with the wave functions of the LLLs. Our deduction of the effective Hamiltonian describing the supplementary, indirect interaction between the particles lying on the LLLs takes into account that two particles during the Coulomb scattering process as a first step perform the virtual quantum transitions to ELLs and as a second step they return back to LLLs. Such quantum transitions depend on the distance between the Landau levels and the supplementary indirect interaction is characterized by a small parameter $I_{\mathrm{ex}}(0)/\hbar\omega_c$ equal to the ration of the magnetoexciton ionization potential $I_{\mathrm{ex}}(0)$ to the Landau quantization energy $\hbar\omega_c$. This parameter decreases as $H^{-1/2}$ with the increasing of the magnetic field strength H. The supplementary electron–electron, hole–hole and electron–hole indirect interactions are attractive. The quantum averages of these terms in Hartree approximation lower the energies of the quasiparticles making, for example, the magnetoexcitons more robust. At the same time their exchange, Fock terms as well as the Bogoliubov u–v transformation terms give rise to a positive, repulsion-type contributions to the chemical potential of the BEC-ed 2D magnetoexcitons with wave vector $k=0$. Their ground state energy and chemical potential obtained in a Hartree–Fock–Bogoliubov approximation (HFBA), have increasing dependence on the filling factor v^2, what stabilize the BEC, permitting to avoid its collapse and to discuss the collective elementary excitations arising in this conditions.

The collective elementary excitations were discussed in both limiting cases. One of them is the Bose-gas of 2D magnetoexcitons in a state of BEC with wave vector $k=0$, and the another one is the metastable dielectric liquid phase (MDLP) formed by BEC-ed magnetoexcitons with wave vectors $kl \sim 3-4$.

The collective elementary excitations were calculated in the frame of the perturbation theory with the infinitesimal parameter $v^2(1-v^2)$ equal to the product of the filling factor v^2 on the phase space filling factor $(1-v^2)$ reflecting the Pauli exclusion principle. The breaking of the gauge symmetry was introduce following the Keldysh–Kozlov–Kopaev (KKK) method[22,23] in combination with the Bogoliubov quasi-average theory approximation.[31] The equations of motion were written for the integral two-particle operators describing electron and hole density fluctuations as well as the creation and annihilation of the magnetoexcitons. The infinite chains of the equations of motion for the Green's functions were truncated following the Zubarev method.[36] The collective elementary excitations in both limiting cases of BEC with $k=0$ and $kl \sim 3-4$ consists from exciton and plasmon branches. There are energy and quasienergy branches related with the existence of the condensate. In the collinear geometry of excitation the four order dispersion equations disintegrates in two equations, one of the third order for the exciton and acoustical plasmon branches and another one for the optical plasmon branches. Side by side with the separate exciton and plasmon branches the mixed exciton—acoustical plasmon as well as two exciton excitations appears, when the self-energy parts containing the frequency in the denominators are taken into account.

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